

Non-relativistic AdS branes and Newton-Hooke superalgebra

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ABSTRACT: We examine a non-relativistic limit of D-branes in $AdS_5 \times S^5$ and M-branes in $AdS_{4/7} \times S^{7/4}$. First, Newton-Hooke superalgebras for the AdS branes are derived from $AdS \times S$ superalgebras as Inönü-Wigner contractions. It is shown that the directions along which the AdS-brane worldvolume extends are restricted by requiring that the isometry on the AdS-brane worldvolume and the Lorentz symmetry in the transverse space naturally extend to the super-isometry. We also derive Newton-Hooke superalgebras for pp-wave branes and show that the directions along which a brane worldvolume extends are restricted. Then the Wess-Zumino terms of the AdS branes are derived by using the Chevalley-Eilenberg cohomology on the super- $AdS \times S$ algebra, and the non-relativistic limit of the AdS-brane actions is considered. We show that the consistent limit is possible for the following branes: Dp (even,even) for $p = 1 \bmod 4$ and Dp (odd,odd) for $p = 3 \bmod 4$ in $AdS_5 \times S^5$, and M2 (0,3), M2 (2,1), M5 (1,5) and M5 (3,3) in $AdS_4 \times S^7$ and $S^4 \times AdS_7$. We furthermore present non-relativistic actions for the AdS branes.

KEYWORDS: Penrose limit and pp-wave background, AdS-CFT Correspondence, p-branes, D-branes.

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1. Introduction

The AdS/CFT conjecture [1] predicts that type IIB superstring theory in $\text{AdS}_5 \times \text{S}^5$ is dual to the four-dimensional $\mathcal{N} = 4 \text{SU}(N)$ super Yang-Mills theory in large N limit. Though it is too hard to analyze the full AdS superstring, Berenstein-Maldacena-Nastase (BMN) found a nice way to extract a solvable subsector (referred to as BMN sector) [2]. Taking this subsector corresponds to the so-called Penrose limit for the AdS geometry [3], and the relevant symmetry to the BMN sector is the pp-wave superalgebra, which is obtained as an Inönü-Wigner (IW) contraction [4] of the super- $\text{AdS}_5 \times \text{S}^5$ algebra [5] (see [6] for the eleven-dimensional cases).

A non-relativistic limit of strings in flat spacetime provides another solvable sector [7] (see also [8]). This limit is a truncation of the full theory in the sense that light states satisfying a Galilean invariant dispersion relation are kept and the rest decouples. The relevant symmetry is the Galilean limit of the Poincaré algebra. The non-relativistic flat branes are examined in [9–13]. In [14, 15] these studies have been extended to branes in AdS spaces. In particular a Lorentzian F-string in $\text{AdS}_5 \times \text{S}^5$, i.e. AdS_2 brane, was examined in [15]. They showed that the F-string theory in $\text{AdS}_5 \times \text{S}^5$ is reduced to a free theory in the non-relativistic limit, and so the resulting theory is exactly solvable. In the non-relativistic limit, the super- $\text{AdS}_5 \times \text{S}^5$ algebra is also contracted to the Newton-Hooke (NH) superalgebra for the F-string. Then the isometry of the AdS_2 -brane worldvolume, the AdS_2 algebra $\text{so}(1,2)$, and the Lorentz symmetry in the transverse space, $\text{so}(3) \times \text{so}(5)$, extend to a super-isometry algebra.

In this paper we consider D-branes in $\text{AdS}_5 \times \text{S}^5$ and M-branes in $\text{AdS}_{4/7} \times \text{S}^{7/4}$. First we examine D-branes in $\text{AdS}_5 \times \text{S}^5$. In addition to AdS_2 brane, there exist various AdS branes in $\text{AdS}_5 \times \text{S}^5$, (m, n) branes of which worldvolume extends along m directions in AdS_5 and n -directions in S^5 . In our previous works [16–19], we have classified some possible configurations of the D-branes in $\text{AdS}_5 \times \text{S}^5$ by examining the κ -variation surface terms of an open superstring. Here we will classify possible configurations of D-branes by requiring that the isometry of the AdS brane worldvolume $\text{AdS}_m \times \text{S}^n$ ($\text{H}^m \times \text{S}^n$) and the Lorentz symmetry

in the transverse space $\mathbb{E}^{5-m} \times \mathbb{E}^{5-n}$ ($\mathbb{E}^{4-m,1} \times \mathbb{E}^{5-n}$), i.e., $\mathfrak{so}(m-1, 2) \times \mathfrak{so}(n+1) \times \mathfrak{so}(5-m) \times \mathfrak{so}(5-n)$ for a Lorentzian brane and $\mathfrak{so}(m, 1) \times \mathfrak{so}(n+1) \times \mathfrak{so}(4-m, 1) \times \mathfrak{so}(5-n)$ for a Euclidean brane, naturally extend to the super-isometry. The result surely contains our previous result, but some new configurations are allowed to exist. We furthermore derive the NH superalgebras for these branes as IW contractions of the super- $\text{AdS}_5 \times \text{S}^5$ algebra. The similar analyses are applied to branes in IIB pp-wave, and derive the NH superalgebras for these branes as IW contractions of the IIB pp-wave superalgebra.

The Wess-Zumino (WZ) terms for p -branes in flat spacetime can be classified [20] as non-trivial elements of the Chevalley-Eilenberg (CE) cohomology [21]. This is generalized to D-branes in [22, 23] by introducing an additional two form which corresponds to a modified field strength of the background B field. Here we examine the WZ terms for AdS branes by using the CE cohomology on \mathfrak{g} of the superspace

$$G = \text{PSU}(2, 2|4)/(\text{SO}(4, 1) \times \text{SO}(5)), \quad \text{i.e.} \quad \text{“super-AdS}_5 \times \text{S}^5\text{”}/\text{“Lorentz”}.$$

We show that the WZ terms of AdS branes can be classified as non-trivial elements of the CE cohomology, except for the WZ term of a string which is a trivial element [24, 25].

Expanding the supercurrents with respect to the scaling used in the IW contraction, we obtain the non-relativistic limit of the brane action. In comparison to the Penrose limit in which the leading terms in the expansion contribute to the pp-wave brane actions (see appendix C), in the non-relativistic limit the leading order terms of the Dirac-Born-Infeld (Nambu-Goto) part and the WZ part cancel out each other, and the next-to-leading order terms contribute to the non-relativistic action. We find that the consistent non-relativistic limit exists only for Dp (even, even) for $p = 1 \pmod{4}$ and Dp (odd, odd) for $p = 3 \pmod{4}$ in $\text{AdS}_5 \times \text{S}^5$. We derive the non-relativistic AdS D-brane action and find that it is reduced to a simple action by fixing the κ -gauge symmetry and the worldvolume reparametrization. While the non-relativistic AdS D-string action is a free field action, the non-relativistic AdS Dp -brane action ($p > 1$) contains an additional term which originates from the flux contribution in the WZ term. The non-relativistic flat D-brane actions obtained in [12] are reproduced as a flat limit of the non-relativistic AdS D-brane actions.

Next we examine a non-relativistic limit of M-branes in $\text{AdS}_{4/7} \times \text{S}^{7/4}$. The NH superalgebra for M-branes are derived as IW contractions of the super- $\text{AdS}_{4/7} \times \text{S}^{7/4}$ algebras. To achieve this, we show that the directions along which a brane worldvolume extends are restricted by requiring that the isometry of the AdS brane worldvolume and the Lorentz symmetry in the transverse space naturally extend to the super-isometry, and that possible M-branes are classified. As expected, the configurations obtained in [26, 27] by examining the κ -variation surface term of an open supermembrane are contained in the above classification. The similar analyses are applied to branes in M pp-wave, and derive the NH superalgebras for these branes as IW contractions of the M pp-wave superalgebra. We obtain the WZ terms of AdS branes as non-trivial elements of the CE cohomology on \mathfrak{g} of the superspace

$$G = \text{OSp}(8|4)/(\text{SO}(3,1) \times \text{SO}(7)) \quad \text{or} \quad \text{OSp}(8^*|4)/(\text{SO}(4) \times \text{SO}(6,1)).$$

We find that the non-relativistic limit exists for M2 (0,3), M2 (2,1), M5 (1,5) and M5 (3,3) in $\text{AdS}_4 \times \text{S}^7$ and $\text{S}^4 \times \text{AdS}_7$. By taking the non-relativistic limit of these AdS brane actions, we derive the non-relativistic M-brane actions in $\text{AdS}_{4/7} \times \text{S}^{7/4}$. It is shown that by fixing the κ -gauge symmetry and the reparametrization the non-relativistic action for AdS M2- and AdS M5-branes is reduced to a simple action which contains an additional term originating from the flux contribution of the WZ term. The non-relativistic flat M2-brane action given in [11] is reproduced as a flat limit of the non-relativistic AdS M2-brane action.

This paper is divided into the two parts. Sections 2-5 are devoted to studies of AdS branes in ten-dimensions, and those in eleven-dimensions are examined in sections 6-9. In section 2, NH superalgebras for branes in $\text{AdS}_5 \times \text{S}^5$ are derived as IW contractions of the super- $\text{AdS}_5 \times \text{S}^5$ algebra. It is shown that the directions along which the AdS brane world-volume extends are restricted by requiring that the isometry on the AdS brane worldvolume and the Lorentz symmetry in the transverse space naturally extend to the super-isometry. The similar analyses are applied to branes in IIB pp-wave in section 3. WZ terms of AdS branes are derived by using the CE cohomology on the $\text{AdS} \times \text{S}$ superalgebra in section 4. Examining a non-relativistic limit of AdS brane actions, we obtain non-relativistic AdS brane actions in section 5. From section 6, M-theory in $\text{AdS}_{4/7} \times \text{S}^{7/4}$ is examined. We derive NH superalgebras for M-branes as IW contractions of the super- $\text{AdS}_{4/7} \times \text{S}^{7/4}$ algebras in section 6. The similar analyses are applied to branes in M pp-wave in section 7. After deriving WZ terms of AdS M-branes by using the CE cohomology on the $\text{AdS}_{4/7} \times \text{S}^{7/4}$ superalgebras in section 8, we examine the non-relativistic limit of AdS M-brane actions in section 9. The last section is devoted to a summary and discussions.

The supervielbeins and the super spin-connections are given in appendix A. In appendix B, the κ -symmetry of Euclidean/Lorentzian brane actions is derived. Our construction of brane actions is applicable to branes in a pp-wave by taking the Penrose limit instead of non-relativistic limit. In fact, we derive brane actions in the pp-wave in appendix C.

2. NH superalgebra of branes in $\text{AdS}_5 \times \text{S}^5$

The super- $\text{AdS}_5 \times \text{S}^5$ algebra, $\text{psu}(2,2|4)$, is generated by translation $P_A = (P_a, P_{a'})$, Lorentz rotation $J_{AB} = (J_{ab}, J_{a'b'})$ and Majorana-Weyl supercharges $Q_I (I = 1, 2)$ as

$$\begin{aligned}
 [P_a, P_b] &= \lambda^2 J_{ab}, & [P_{a'}, P_{b'}] &= -\lambda^2 J_{a'b'}, \\
 [P_a, J_{bc}] &= \eta_{ab} P_c - \eta_{ac} P_b, & [P_{a'}, J_{b'c'}] &= \eta_{a'b'} P_{c'} - \eta_{a'c'} P_{b'}, \\
 [J_{ab}, J_{cd}] &= \eta_{bc} J_{ad} + \text{3-terms}, & [J_{a'b'}, J_{c'd'}] &= \eta_{b'c'} J_{a'd'} + \text{3-terms}, \\
 [Q_I, P_A] &= -\frac{\lambda}{2} Q_J (i\sigma_2)_{JI} \hat{\Gamma}_A, & [Q_I, J_{AB}] &= -\frac{1}{2} Q_I \Gamma_{AB}, \\
 \{Q_I, Q_J\} &= 2i\mathcal{C} \Gamma^A \delta_{IJ} h_+ P_A - i\lambda \mathcal{C} \hat{\Gamma}^{AB} (i\sigma_2)_{IJ} h_+ J_{AB},
 \end{aligned} \tag{2.1}$$

where $a = 0, \dots, 4$ and $a' = 5, \dots, 9$ are vector indices of AdS_5 and S^5 respectively. The gamma matrix $\Gamma^A \in \text{Spin}(1,9)$ satisfies

$$\{\Gamma^A, \Gamma^B\} = 2\eta^{AB}, \quad (\Gamma^A)^T = -\mathcal{C} \Gamma^A \mathcal{C}^{-1}, \quad \mathcal{C}^T = -\mathcal{C} \tag{2.2}$$

where \mathcal{C} is the charge conjugation matrix. We use almost positive Minkowski metric η_{AB} and define

$$\begin{aligned}\widehat{\Gamma}_A &= (-\Gamma_a \mathcal{I}, \Gamma_{a'} \mathcal{J}), & \widehat{\Gamma}_{AB} &= (-\Gamma_{ab} \mathcal{I}, \Gamma_{a'b'} \mathcal{J}), & \mathcal{I} &= \Gamma^{01234}, & \mathcal{J} &= \Gamma^{56789}, \\ Q_I h_+ &= Q_I, & h_+ &= \frac{1}{2}(1 + \Gamma_{11}), & \Gamma_{11} &= \Gamma_{01\dots 9},\end{aligned}\tag{2.3}$$

and $\lambda = 1/R$ where R is the radii of AdS_5 and S^5 .

By using an element $g \in \text{PSU}(2,2|4)$, a left-invariant (LI) Cartan one-form is defined as

$$\Omega = g^{-1} dg \equiv \mathbf{L}^A P_A + \frac{1}{2} \mathbf{L}^{AB} J_{AB} + Q_I L^I.\tag{2.4}$$

Then the Maurer-Cartan (MC) equation, which is satisfied by LI Cartan one-forms

$$dL^{\hat{A}} = \frac{1}{2} L^{\hat{B}} L^{\hat{C}} f_{\hat{C}\hat{B}}^{\hat{A}}, \quad \Omega = L^{\hat{A}} T_{\hat{A}},\tag{2.5}$$

is equivalent to the superalgebra $[T_{\hat{A}}, T_{\hat{B}}] = f_{\hat{A}\hat{B}}^{\hat{C}} T_{\hat{C}}$. The Jacobi identities $f_{[\hat{A}\hat{B}}^{\hat{D}} f_{|\hat{D}|\hat{C}}^{\hat{E}} = 0$ of the commutation relation of the superalgebra is stated as the nilpotency of the differential, $d^2 = 0$. Thus (2.1) is equivalent to

$$\begin{aligned}d\mathbf{L}^A &= -\eta_{BC} \mathbf{L}^{AB} \mathbf{L}^C + i\bar{L} \Gamma^A L, \\ d\mathbf{L}^{ab} &= -\lambda^2 \mathbf{L}^a \mathbf{L}^b - \eta_{cd} \mathbf{L}^{ca} \mathbf{L}^{bd} + i\lambda \bar{L} \Gamma^{ab} \mathcal{I} i\sigma_2 L, \\ d\mathbf{L}^{a'b'} &= +\lambda^2 \mathbf{L}^{a'} \mathbf{L}^{b'} - \eta_{c'd'} \mathbf{L}^{c'a'} \mathbf{L}^{b'd'} - i\lambda \bar{L} \Gamma^{a'b'} \mathcal{J} i\sigma_2 L, \\ dL^\alpha &= -\frac{\lambda}{2} \mathbf{L}^A \widehat{\Gamma}_A i\sigma_2 L - \frac{1}{4} \mathbf{L}^{AB} \Gamma_{AB} L.\end{aligned}\tag{2.6}$$

We derive NH superalgebras for AdS branes as IW contractions of the super- $\text{AdS}_5 \times \text{S}^5$ algebra.

First we consider the bosonic subalgebra. Let us introduce the following coordinates:

$$\bar{A} = A_0, \dots, A_p, \quad \underline{A} = A_{p+1}, \dots, A_9,\tag{2.7}$$

where $\bar{A} = (\bar{a}, \bar{a}')$ represent the worldvolume directions of the AdS brane. When the worldvolume extends along m directions in AdS_5 and n directions in S^5 , we call it an (m, n) -brane. We rescale the generators as follows:

$$P_{\underline{A}} \rightarrow \frac{1}{\Omega} P_{\underline{A}}, \quad J_{\bar{A}\bar{B}} \rightarrow \frac{1}{\Omega} J_{\bar{A}\bar{B}}.\tag{2.8}$$

The limit $\Omega \rightarrow 0$ leads to the NH algebra for the AdS brane

$$\begin{aligned}[P_{\bar{a}}, P_{\bar{b}}] &= \lambda^2 J_{\bar{a}\bar{b}}, & [P_{\bar{a}'}, P_{\bar{b}'}] &= -\lambda^2 J_{\bar{a}'\bar{b}'}, \\ [P_{\bar{a}}, P_{\bar{b}}] &= \lambda^2 J_{\bar{a}\bar{b}}, & [P_{\bar{a}'}, P_{\bar{b}'}] &= -\lambda^2 J_{\bar{a}'\bar{b}'}, \\ [P_{\bar{A}}, J_{\bar{B}\bar{C}}] &= \eta_{\bar{A}\bar{B}} P_{\bar{C}} - \eta_{\bar{A}\bar{C}} P_{\bar{B}}, & [P_{\underline{A}}, J_{\underline{B}\underline{C}}] &= \eta_{\underline{A}\underline{B}} P_{\underline{C}} - \eta_{\underline{A}\underline{C}} P_{\underline{B}}, \\ [P_{\bar{A}}, J_{\bar{B}\bar{C}}] &= \eta_{\bar{A}\bar{B}} P_{\bar{C}}, \\ [J_{\bar{A}\bar{B}}, J_{\bar{C}\bar{D}}] &= \eta_{\bar{A}\bar{D}} J_{\bar{B}\bar{C}} + 3\text{-terms}, & [J_{\underline{A}\underline{B}}, J_{\underline{C}\underline{D}}] &= \eta_{\underline{A}\underline{D}} J_{\underline{B}\underline{C}} + 3\text{-terms}, \\ [J_{\bar{A}\bar{B}}, J_{\bar{C}\bar{D}}] &= \eta_{\bar{B}\bar{C}} J_{\bar{A}\bar{D}} - \eta_{\bar{A}\bar{C}} J_{\bar{B}\bar{D}}, & [J_{\underline{A}\underline{B}}, J_{\underline{C}\underline{D}}] &= \eta_{\underline{B}\underline{D}} J_{\underline{C}\underline{A}} - \eta_{\underline{A}\underline{D}} J_{\underline{C}\underline{B}}.\end{aligned}\tag{2.9}$$

ρ	1-brane	3-brane	5-brane	7-brane	9-brane
σ_1, σ_3	(2,0), (0,2)		(4,2), (2,4)		
$i\sigma_2$		(3,1), (1,3)		(5,3), (3,5)	
1	(1,1)		(5,1), (3,3), (1,5)		(5,5)

Table 1: Branes in $\text{AdS}_5 \times \text{S}^5$

This is the NH algebra of a brane given in [14] (see also [28]). The NH algebra contains two subalgebras. One is the isometry of (m, n) -brane worldvolume generated by $\{P_{\bar{A}}, J_{\bar{A}\bar{B}}\}$, the $\text{AdS}_m \times \text{S}^n$ algebra $\text{so}(m-1, 2) \times \text{so}(n+1)$ for a Lorentzian brane and the $H^m \times \text{S}^n$ algebra $\text{so}(m, 1) \times \text{so}(n+1)$ for a Euclidean brane. The other is the Poincaré algebra, $\text{iso}(5-m) \times \text{iso}(5-n)$ for a Lorentzian brane and $\text{iso}(4-m, 1) \times \text{iso}(5-n)$ for a Euclidean brane, generated by $\{P_{\underline{A}}, J_{\underline{A}\underline{B}}\}$ which is the isometry of the transverse space $\mathbb{E}^{5-m} \times \mathbb{E}^{5-n}$ and $\mathbb{E}^{4-m, 1} \times \mathbb{E}^{5-n}$ respectively.

Next, we consider the fermionic part. Let us introduce a condition

$$\theta = M\theta \quad \text{with} \quad M = \ell \Gamma^{\bar{A}_0 \dots \bar{A}_p} \otimes \rho \quad (2.10)$$

where $\ell^2(-1)^{\lfloor \frac{p+1}{2} \rfloor} \rho^2 = 1$ for $M^2 = 1$. The 2×2 matrix ρ is determined below. As $\theta = h_+ \theta$, $[M, h_+] = 0$ is required so that $p = \text{odd}$. We demand that M satisfies following relations

$$M' \Gamma^{\bar{A}} = \Gamma^{\bar{A}} M, \quad (2.11)$$

$$M' \hat{\Gamma}^{\bar{A}\bar{B}} i\sigma_2 = \hat{\Gamma}^{\bar{A}\bar{B}} i\sigma_2 M, \quad (2.12)$$

where $M' = C^{-1} M^T C$. If these are satisfied, the isometry of the AdS brane worldvolume and the Lorentz symmetry in the transverse space, $\text{so}(m-1, 2) \times \text{so}(n+1) \times \text{so}(5-m) \times \text{so}(5-n)$ for a Lorentzian brane and $\text{so}(m, 1) \times \text{so}(n+1) \times \text{so}(4-m, 1) \times \text{so}(5-n)$ for a Euclidean brane, naturally extend to the super-isometry as will be seen below. It is straightforward to see that the first condition is satisfied by $\rho^T = \rho$ for $p = 1 \pmod{4}$ and by $\rho^T = -\rho$ for $p = 3 \pmod{4}$. The second condition restricts the direction along which branes extend. Since, for $\rho = 1$ ($p = 1 \pmod{4}$) and $\rho = i\sigma_2$ ($p = 3 \pmod{4}$), we derive

$$M' \hat{\Gamma}^{\bar{A}\bar{B}} i\sigma_2 = (-1)^d \hat{\Gamma}^{\bar{A}\bar{B}} i\sigma_2 M, \quad (2.13)$$

we have (odd, odd)-branes. d denotes the number of Dirichlet directions contained in AdS_5 . On the other hand, for $\rho = \sigma_1, \sigma_3$ ($p = 1 \pmod{4}$), since

$$M' \hat{\Gamma}^{\bar{A}\bar{B}} i\sigma_2 = -(-1)^d \hat{\Gamma}^{\bar{A}\bar{B}} i\sigma_2 M, \quad (2.14)$$

(even, even)-branes are allowed. In both cases, we have $\ell = \sqrt{-s}$ and

$$M' = -M. \quad (2.15)$$

We summarize branes in table 1. The 9-brane is nothing but $\text{AdS}_5 \times \text{S}^5$ itself as $M = h_+$ in this case. This table shows possible 1/2 supersymmetric subspaces which are not necessarily

1/2 BPS Dirichlet branes. The (even,even)-branes with $p = 1 \pmod 4$ and (odd,odd)-branes with $p = 3 \pmod 4$ are 1/2 BPS Dirichlet branes of F- and D-strings in $\text{AdS}_5 \times S^5$ derived in [16–18],¹ while (odd,odd)-branes with $p = 1 \pmod 4$ are not 1/2 BPS Dirichlet branes. In the presence of gauge field condensates, see [19]. As will be seen in section 4, we find consistent non-relativistic limits for 1/2 BPS AdS branes.

Let us decompose Q^α with the projection operator

$$\mathcal{P}_\pm = \frac{1}{2}(1 \pm M) \quad \text{as} \quad Q = Q_+ + Q_-, \quad Q_\pm \mathcal{P}_\pm = Q_\pm, \quad (2.16)$$

and rescale fermionic generators as

$$Q_+ \rightarrow Q_+, \quad Q_- \rightarrow \frac{1}{\Omega} Q_- . \quad (2.17)$$

Taking $\Omega \rightarrow 0$ leads to (anti-)commutation relations

$$\begin{aligned} [P_{\bar{A}}, Q_+] &= \frac{\lambda}{2} Q_+ \hat{\Gamma}_{\bar{A}} i\sigma_2, & [P_{\underline{A}}, Q_+] &= \frac{\lambda}{2} Q_- \hat{\Gamma}_{\underline{A}} i\sigma_2, & [P_{\bar{A}}, Q_-] &= \frac{\lambda}{2} Q_- \hat{\Gamma}_{\bar{A}} i\sigma_2, \\ [J_{\bar{A}\bar{B}}, Q_\pm] &= \frac{1}{2} Q_\pm \Gamma_{\bar{A}\bar{B}}, & [J_{\underline{A}\underline{B}}, Q_\pm] &= \frac{1}{2} Q_\pm \Gamma_{\underline{A}\underline{B}}, & [J_{\bar{A}\bar{B}}, Q_+] &= \frac{1}{2} Q_- \Gamma_{\bar{A}\bar{B}}, \\ \{Q_+, Q_+\} &= 2i\mathcal{C}\Gamma^{\bar{A}} h_+ \mathcal{P}_+ P_{\bar{A}} - i\lambda\mathcal{C}\hat{\Gamma}^{\bar{A}\bar{B}} i\sigma_2 h_+ \mathcal{P}_+ J_{\bar{A}\bar{B}} - i\lambda\mathcal{C}\hat{\Gamma}^{\underline{A}\underline{B}} i\sigma_2 h_+ \mathcal{P}_+ J_{\underline{A}\underline{B}}, \\ \{Q_+, Q_-\} &= 2i\mathcal{C}\Gamma^{\bar{A}} h_+ \mathcal{P}_- P_{\bar{A}} - 2i\lambda\mathcal{C}\hat{\Gamma}^{\bar{A}\bar{B}} i\sigma_2 h_+ \mathcal{P}_- J_{\bar{A}\bar{B}} . \end{aligned} \quad (2.18)$$

In summary, we have derived the NH superalgebra for AdS brane, (2.9) and (2.18), as an IW contraction of $\text{psu}(2,2|4)$. The NH superalgebra for an F-string [15] is contained as the $p = 1$ case.

We note that generators $P_{\bar{A}}, J_{\bar{A}\bar{B}}, J_{\underline{A}\underline{B}}$ and Q_+ form a super-subalgebra

$$\begin{aligned} [P_{\bar{a}}, P_{\bar{b}}] &= \lambda^2 J_{\bar{a}\bar{b}}, & [P_{\bar{a}'}, P_{\bar{b}'}] &= -\lambda^2 J_{\bar{a}'\bar{b}'}, & [P_{\bar{A}}, J_{\bar{B}\bar{C}}] &= \eta_{\bar{A}\bar{B}} P_{\bar{C}} - \eta_{\bar{A}\bar{C}} P_{\bar{B}}, \\ [J_{\bar{A}\bar{B}}, J_{\bar{C}\bar{D}}] &= \eta_{\bar{A}\bar{D}} J_{\bar{B}\bar{C}} + \text{3-terms}, & [J_{\underline{A}\underline{B}}, J_{\underline{C}\underline{D}}] &= \eta_{\underline{A}\underline{D}} J_{\underline{B}\underline{C}} + \text{3-terms}, \\ [P_{\bar{A}}, Q_+] &= \frac{\lambda}{2} Q_+ \hat{\Gamma}_{\bar{A}} i\sigma_2, & [J_{\bar{A}\bar{B}}, Q_+] &= \frac{1}{2} Q_+ \Gamma_{\bar{A}\bar{B}}, & [J_{\underline{A}\underline{B}}, Q_+] &= \frac{1}{2} Q_+ \Gamma_{\underline{A}\underline{B}}, \\ \{Q_+, Q_+\} &= 2i\mathcal{C}\Gamma^{\bar{A}} h_+ \mathcal{P}_+ P_{\bar{A}} - i\lambda\mathcal{C}\hat{\Gamma}^{\bar{A}\bar{B}} i\sigma_2 h_+ \mathcal{P}_+ J_{\bar{A}\bar{B}} - i\lambda\mathcal{C}\hat{\Gamma}^{\underline{A}\underline{B}} i\sigma_2 h_+ \mathcal{P}_+ J_{\underline{A}\underline{B}}, \end{aligned} \quad (2.19)$$

which is a supersymmetrization of $\text{so}(m-1, 2) \times \text{so}(n+1) \times \text{so}(5-m) \times \text{so}(5-n)$ for a Lorentzian brane and $\text{so}(m, 1) \times \text{so}(n+1) \times \text{so}(4-m, 1) \times \text{so}(5-n)$ for a Euclidean brane. The superalgebra for the (5,5)-brane is $\text{psu}(2,2|4)$. Since the dimension of the bosonic subalgebra is 14 for (1,1)-, (3,1)-, (1,3)- and (3,3)-branes, 16 for (2,0)-, (0,2)-, (4,2)-, (2,4)-branes, and 22 for (5,1)-, (1,5)-, (5,3)- and (3,5)-branes, one may guess the corresponding superalgebras as those including variants of $\text{su}(2|2) \times \text{su}(2|2)$, $\text{osp}(4|4)$ and $\text{osp}(6|2) \times \text{psu}(2|1)$, respectively. The existence of these superalgebras is ensured by (2.11) and (2.12).

¹The brane probe analysis [29] is also consistent with this result.

It is straightforward to derive MC equations for the AdS brane NH superalgebra (2.9) and (2.18)

$$d\mathbf{L}^{\bar{A}} = -\eta_{\bar{B}\bar{C}}\mathbf{L}^{\bar{A}\bar{B}}\mathbf{L}^{\bar{C}} + i\bar{L}_+\Gamma^{\bar{A}}L_+, \quad (2.20)$$

$$d\mathbf{L}^A = -\eta_{\bar{B}\bar{C}}\mathbf{L}^{\bar{A}\bar{B}}\mathbf{L}^{\bar{C}} - \eta_{BC}\mathbf{L}^{AB}\mathbf{L}^C + i\bar{L}_+\Gamma^AL_- + i\bar{L}_-\Gamma^AL_+, \quad (2.21)$$

$$d\mathbf{L}^{\bar{a}\bar{b}} = -\lambda^2\mathbf{L}^{\bar{a}}\mathbf{L}^{\bar{b}} - \eta_{\bar{c}\bar{d}}\mathbf{L}^{\bar{c}\bar{a}}\mathbf{L}^{\bar{b}\bar{d}} - i\lambda\bar{L}_+\widehat{\Gamma}^{\bar{a}\bar{b}}i\sigma_2L_+, \quad (2.22)$$

$$d\mathbf{L}^{\bar{a}'\bar{b}'} = +\lambda^2\mathbf{L}^{\bar{a}'}\mathbf{L}^{\bar{b}'} - \eta_{\bar{c}'\bar{d}'}\mathbf{L}^{\bar{c}'\bar{a}'}\mathbf{L}^{\bar{b}'\bar{d}'} - i\lambda\bar{L}_+\widehat{\Gamma}^{\bar{a}'\bar{b}'}i\sigma_2L_+, \quad (2.23)$$

$$d\mathbf{L}^{AB} = -\eta_{CD}\mathbf{L}^{CA}\mathbf{L}^{BD} - i\lambda\bar{L}_+\widehat{\Gamma}^{AB}i\sigma_2L_+, \quad (2.24)$$

$$d\mathbf{L}^{\bar{a}b} = -\lambda^2\mathbf{L}^{\bar{a}}\mathbf{L}^b - \eta_{\bar{c}\bar{d}}\mathbf{L}^{\bar{c}\bar{a}}\mathbf{L}^{b\bar{d}} - \eta_{cd}\mathbf{L}^{c\bar{a}}\mathbf{L}^{bd} \\ - i\lambda\bar{L}_+\widehat{\Gamma}^{\bar{a}b}i\sigma_2L_- - i\lambda\bar{L}_-\widehat{\Gamma}^{\bar{a}b}i\sigma_2L_+, \quad (2.25)$$

$$d\mathbf{L}^{\bar{a}'b'} = +\lambda^2\mathbf{L}^{\bar{a}'}\mathbf{L}^{b'} - \eta_{\bar{c}'\bar{d}'}\mathbf{L}^{\bar{c}'\bar{a}'}\mathbf{L}^{b'\bar{d}'} - \eta_{c'd'}\mathbf{L}^{c'\bar{a}'}\mathbf{L}^{b'd'} \\ - i\lambda\bar{L}_+\widehat{\Gamma}^{\bar{a}'b'}i\sigma_2L_- - i\lambda\bar{L}_-\widehat{\Gamma}^{\bar{a}'b'}i\sigma_2L_+, \quad (2.26)$$

$$dL_+ = -\frac{\lambda}{2}\mathbf{L}^{\bar{A}}\widehat{\Gamma}_{\bar{A}}i\sigma_2L_+ - \frac{1}{4}\mathbf{L}^{\bar{A}\bar{B}}\Gamma_{\bar{A}\bar{B}}L_+ - \frac{1}{4}\mathbf{L}^{AB}\Gamma_{AB}L_+, \quad (2.27)$$

$$dL_- = -\frac{\lambda}{2}\mathbf{L}^{\bar{A}}\widehat{\Gamma}_{\bar{A}}i\sigma_2L_- - \frac{\lambda}{2}\mathbf{L}^A\widehat{\Gamma}_Ai\sigma_2L_+ \\ - \frac{1}{4}\mathbf{L}^{\bar{A}\bar{B}}\Gamma_{\bar{A}\bar{B}}L_- - \frac{1}{4}\mathbf{L}^{AB}\Gamma_{AB}L_- - \frac{1}{2}\mathbf{L}^{\bar{A}\bar{B}}\Gamma_{\bar{A}\bar{B}}L_+. \quad (2.28)$$

An alternative way to derive these MC equations is to rescale the Cartan one-forms in the MC equation (2.6) as

$$\mathbf{L}^A \rightarrow \Omega\mathbf{L}^A, \quad \mathbf{L}^{\bar{A}\bar{B}} \rightarrow \Omega\mathbf{L}^{\bar{A}\bar{B}}, \quad L_- \rightarrow \Omega L_- \quad (2.29)$$

and take the limit $\Omega \rightarrow 0$. This provides the leading order terms of the expansion considered in the non-relativistic limit in section 4.

Finally, let us consider an alternative scaling

$$\lambda \rightarrow \frac{1}{\omega}\lambda, \quad P_{\bar{A}} \rightarrow \frac{1}{\omega}P_{\bar{A}}, \quad J_{\bar{A}\bar{B}} \rightarrow \omega J_{\bar{A}\bar{B}}, \quad Q_+ \rightarrow \frac{1}{\sqrt{\omega}}Q_+, \quad Q_- \rightarrow \sqrt{\omega}Q_- . \quad (2.30)$$

Since λ is absorbed as

$$P_{\bar{A}} \rightarrow \frac{1}{\lambda}P_{\bar{A}}, \quad P_A \rightarrow \frac{1}{\lambda}P_A, \quad Q_{\pm} \rightarrow \frac{1}{\sqrt{\lambda}}Q_{\pm}, \quad (2.31)$$

this is equivalent to (2.8) and (2.17) with $\Omega = 1/\omega$. In this paper, we use (2.8) and (2.17) instead of (2.30), though both limits lead to the same results.

3. NH superalgebra of branes in IIB PP-wave

Type IIB PP-wave superalgebra is obtained as an IW contraction of the super-AdS₅×S⁵ algebra. First of all, let us introduce the following quantities for later convenience,

$$P_{\pm} = \frac{1}{\sqrt{2}}(P_9 \pm P_0), \quad P_i^* = (P_i^* = J_{0i}, P_{i'}^* = J_{9i'}), \quad (3.1)$$

$$Q = Q^{(+)} + Q^{(-)}, \quad Q^{(\pm)} = Q^{(\pm)}\ell_{\pm}, \quad \ell_{\pm} = \frac{1}{2}\Gamma_{\pm}\Gamma_{\mp}, \quad \Gamma_{\pm} = \frac{1}{\sqrt{2}}(\Gamma_9 \pm \Gamma_0)$$

where $i = 1, 2, 3, 4$ and $i' = 5, 6, 7, 8$. The IW contraction is performed in [5] by scaling generators in the super-AdS₅×S⁵ algebra as

$$P_+ \rightarrow \frac{1}{\Lambda^2} P_+, \quad P_i \rightarrow \frac{1}{\Lambda} P_i, \quad P_i^* \rightarrow \frac{1}{\Lambda} P_i^*, \quad Q^{(+)} \rightarrow \frac{1}{\Lambda} Q^{(+)}, \quad (3.2)$$

and then taking the limit $\Lambda \rightarrow 0$. After the contraction, the super-AdS₅×S⁵ algebra is reduced to the IIB pp-wave superalgebra

$$\begin{aligned} [P_-, P_i] &= -\frac{\lambda^2}{\sqrt{2}} P_i^*, & [P_-, P_i^*] &= \frac{1}{\sqrt{2}} P_i, & [P_i, P_j^*] &= -\frac{1}{\sqrt{2}} \eta_{ij} P_+, \\ [P_i, J_{\hat{j}\hat{k}}] &= \eta_{ij} P_{\hat{k}} - \eta_{i\hat{k}} P_{\hat{j}}, & [P_i^*, J_{\hat{j}\hat{k}}] &= \eta_{ij} P_{\hat{k}}^* - \eta_{i\hat{k}} P_{\hat{j}}^*, & [J_{\hat{i}\hat{j}}, J_{\hat{k}\hat{l}}] &= \eta_{\hat{j}\hat{k}} J_{\hat{i}\hat{l}} + \text{3-terms}, \\ [Q^{(-)}, P_i] &= \frac{1}{2} Q^{(+)} \Gamma_i \mathcal{I} \sigma_2, & [Q^{(+)}, P_-] &= -\frac{1}{2} Q^{(+)} \Gamma_+ \mathcal{I} \sigma_2, \\ [Q^{(-)}, P_i^*] &= \frac{1}{2\sqrt{2}} Q^{(+)} \Gamma_+ \Gamma_i, & [Q^{(\pm)}, J_{\hat{i}\hat{j}}] &= -\frac{1}{2} Q^{(\pm)} \Gamma_{\hat{i}\hat{j}}, \\ \{Q^{(+)}, Q^{(+)}\} &= 2i\mathcal{C}\Gamma_- P_+, \\ \{Q^{(-)}, Q^{(-)}\} &= 2i\mathcal{C}\Gamma_+ P_- - i\frac{\lambda}{\sqrt{2}} \mathcal{C}\hat{\Gamma}^{\hat{i}\hat{j}} i\sigma_2 J_{\hat{i}\hat{j}}, \\ \{Q^{(\pm)}, Q^{(\mp)}\} &= 2i\mathcal{C}\Gamma^{\hat{i}} \ell_{\mp} P_i + i\lambda \mathcal{C}\hat{\Gamma}^{\hat{i}} \ell_{\mp} i\sigma_2 P_i^*, \end{aligned} \quad (3.3)$$

where $\hat{\Gamma}^{\hat{i}\hat{j}} = (-\Gamma^{ij}\Gamma_+ f, \Gamma^{i'j'}\Gamma_+ g)$, $\hat{\Gamma}^{\hat{i}} = (\Gamma^i f, \Gamma^{i'} g)$, $f = \Gamma^{1234}$ and $g = \Gamma^{5678}$. The bosonic subalgebra, the pp-wave algebra, is the semi-direct product of the Heisenberg algebra generated by $\{P_i, P_i^*\}$ with an outer automorphism P_- and the Lorentz algebra generated by $J_{\hat{i}\hat{j}}$.

3.1 Lorentzian branes

Here we consider the case that $(+, -)$ are contained in the Neumann directions. Let us denote the Neumann and the Dirichlet directions, respectively, as

$$\bar{A} = (+, -, \hat{i}), \quad \underline{A} = \hat{i}. \quad (3.4)$$

We derive the NH superalgebra of a Lorentzian pp-wave brane as an IW contraction of the pp-wave superalgebra.

Let us first consider the bosonic subalgebra. We rescale generators in the pp-wave algebra as

$$P_{\underline{A}} \rightarrow \frac{1}{\Omega} P_{\underline{A}}, \quad J_{\hat{i}\hat{j}} \rightarrow \frac{1}{\Omega} J_{\hat{i}\hat{j}}, \quad P_{\hat{i}}^* \rightarrow \frac{1}{\Omega} P_{\hat{i}}^*, \quad (3.5)$$

and then take the limit $\Omega \rightarrow 0$. The resulting algebra is the NH algebra of a pp-wave brane

$$\begin{aligned} [P_-, P_i] &= -\frac{\lambda^2}{\sqrt{2}} P_i^*, & [P_-, P_{\hat{i}}] &= -\frac{\lambda^2}{\sqrt{2}} P_{\hat{i}}^*, & [P_-, P_i^*] &= \frac{1}{\sqrt{2}} P_i, & [P_-, P_{\hat{i}}^*] &= \frac{1}{\sqrt{2}} P_{\hat{i}}, \\ [P_i, P_i^*] &= -\frac{1}{\sqrt{2}} \eta_{\hat{i}\hat{j}} P_+, & [P_i, J_{\hat{j}\hat{k}}] &= \eta_{ij} P_{\hat{k}}, & [J_{\hat{i}\hat{j}}, P_{\hat{k}}^*] &= -\eta_{i\hat{k}} P_{\hat{j}}^*, \end{aligned} \quad (3.6)$$

and

$$\begin{aligned}
 [P_{\hat{i}}, J_{\hat{j}\hat{k}}] &= \eta_{\hat{i}\hat{j}} P_{\hat{k}} - \eta_{\hat{i}\hat{k}} P_{\hat{j}}, & [P_{\hat{l}}, J_{\hat{j}\hat{k}}] &= \eta_{\hat{l}\hat{j}} P_{\hat{k}} - \eta_{\hat{l}\hat{k}} P_{\hat{j}}, \\
 [J_{\hat{i}\hat{j}}, P_{\hat{k}}^*] &= \eta_{\hat{j}\hat{k}} P_{\hat{i}}^* - \eta_{\hat{i}\hat{k}} P_{\hat{j}}^*, & [J_{\hat{i}\hat{j}}, P_{\hat{k}}^*] &= \eta_{\hat{j}\hat{k}} P_{\hat{i}}^* - \eta_{\hat{i}\hat{k}} P_{\hat{j}}^*, \\
 [J_{\hat{i}\hat{j}}, J_{\hat{k}\hat{l}}] &= \eta_{\hat{j}\hat{k}} J_{\hat{i}\hat{l}} + 3\text{-terms}, & [J_{\hat{i}\hat{j}}, J_{\hat{k}\hat{l}}] &= \eta_{\hat{j}\hat{k}} J_{\hat{i}\hat{l}} + 3\text{-terms}, \\
 [J_{\hat{i}\hat{j}}, J_{\hat{k}\hat{l}}] &= \eta_{\hat{j}\hat{k}} J_{\hat{i}\hat{l}} - \eta_{\hat{i}\hat{k}} J_{\hat{j}\hat{l}}, & [J_{\hat{i}\hat{j}}, J_{\hat{k}\hat{l}}] &= \eta_{\hat{i}\hat{l}} J_{\hat{j}\hat{k}} - \eta_{\hat{j}\hat{l}} J_{\hat{i}\hat{k}}.
 \end{aligned} \tag{3.7}$$

Next we consider the fermionic part. We introduce a matrix M

$$M = \ell \Gamma^{+\bar{A}_1 \dots \bar{A}_{p-1}} \rho \tag{3.8}$$

where ρ is a 2×2 matrix. Then $Q^{(\pm)}$ are decomposed into the two parts as follows:

$$Q_{\pm}^{(\bullet)} = \pm Q_{\pm}^{(\bullet)} M. \tag{3.9}$$

The chirality of $Q^{(\bullet)}$ is preserved only when $p = \text{odd}$. In addition, requiring that $M^2 = 1$, we obtain the following condition,

$$\ell^2 (-1)^{\lfloor \frac{p-1}{2} \rfloor} \rho^2 = 1. \tag{3.10}$$

Then we demand that

$$M' \Gamma^{\bar{A}} = \Gamma^{\bar{A}} M, \tag{3.11}$$

$$M' \hat{\Gamma}^{\hat{i}\hat{j}} i\sigma_2 = \hat{\Gamma}^{\hat{i}\hat{j}} i\sigma_2 M \tag{3.12}$$

where

$$M' = \mathcal{C}^{-1} M^T \mathcal{C} = \pm (-1)^{p + \lfloor \frac{p-1}{2} \rfloor} M, \quad \rho^T = \pm \rho. \tag{3.13}$$

Since

$$M' \Gamma^{\bar{A}} = \pm (-1)^{\lfloor \frac{p-1}{2} \rfloor} \Gamma^{\bar{A}} M, \quad \rho^T = \pm \rho, \tag{3.14}$$

the first condition is satisfied by

$$\pm (-1)^{\lfloor \frac{p-1}{2} \rfloor} = 1, \quad \rho^T = \pm \rho. \tag{3.15}$$

This implies that $\rho^T = \rho$ for $p = 1 \pmod{4}$ and $\rho^T = -\rho$ for $p = 3 \pmod{4}$, and that $M' = -M$ and $\ell = 1$. The second condition is rewritten as

$$\pm (-1)^n = 1, \quad \rho^T = \begin{cases} 1, i\sigma_2 \\ \sigma_1, \sigma_3 \end{cases} \tag{3.16}$$

where n is the number of the Neumann directions contained in $\{1, 2, 3, 4\}$ and $\{5, 6, 7, 8\}$ so that the directions along which a pp-wave brane worldvolume extends are restricted. We summarize the results in table 2. This shows possible $1/2$ supersymmetric subspaces which are not necessarily Dirichlet branes of a string. In fact, $(+, -; \text{odd}, \text{odd})$ -branes with

ρ	1-brane	3-brane	5-brane	7-brane	9-brane
σ_1, σ_3			(+, -; 1, 3) (+, -; 3, 1)		
$i\sigma_2$		(+, -; 0, 2) (+, -; 2, 0)		(+, -; 4, 2) (+, -; 2, 4)	
1	(+, -)		(+, -; 0, 4) (+, -; 2, 2) (+, -; 4, 0)		(+, -; 4, 4)

Table 2: Lorentzian pp-wave branes.

$p = 1 \pmod 4$ and (+, -; even, even)-branes with $p = 3 \pmod 4$ are 1/2 BPS D-branes of an open pp-wave superstring [30, 16], while (+, -; even, even)-branes with $p = 1 \pmod 4$ are not. Our results are consistent with those obtained in the brane probe analysis [29], the supergravity analysis [31] and the CFT analysis in the light-cone gauge [32–35].

Scaling $Q_{\pm}^{(\bullet)}$ as

$$Q_+^{(\bullet)} \rightarrow Q_+^{(\bullet)}, \quad Q_-^{(\bullet)} \rightarrow \frac{1}{\Omega} Q_-^{(\bullet)} \quad (3.17)$$

and taking the limit $\Omega \rightarrow 0$, we obtain the fermionic part of the NH superalgebra

$$\begin{aligned}
 [Q_{\pm}^{(-)}, P_{\hat{i}}] &= -\frac{1}{2\sqrt{2}} Q_{\pm}^{(+)} \Gamma_{\hat{i}} \Gamma_+ f i \sigma_2, & [Q_+^{(-)}, P_{\hat{i}}] &= -\frac{1}{2\sqrt{2}} Q_-^{(+)} \Gamma_{\hat{i}} \Gamma_+ f i \sigma_2, \\
 [Q_{\pm}^{(+)}, P_-] &= -\frac{1}{\sqrt{2}} Q_{\pm}^{(+)} f i \sigma_2, & [Q_{\pm}^{(-)}, P_{\hat{i}}^*] &= \frac{1}{2\sqrt{2}} Q_{\pm}^{(+)} \Gamma_+ \Gamma_{\hat{i}}, \\
 [Q_+^{(-)}, P_{\hat{i}}] &= \frac{1}{2\sqrt{2}} Q_-^{(+)} \Gamma_+ \Gamma_{\hat{i}}, & [Q_{\pm}^{(\bullet)}, J_{\hat{i}\hat{j}}] &= -\frac{1}{2} Q_{\pm}^{(\bullet)} \Gamma_{\hat{i}\hat{j}}, & [Q_{\pm}^{(\bullet)}, J_{\hat{i}\hat{j}}] &= -\frac{1}{2} Q_{\pm}^{(\bullet)} \Gamma_{\hat{i}\hat{j}}, \\
 [Q_+^{(\pm)}, J_{\hat{i}\hat{j}}] &= -\frac{1}{2} Q_-^{(\pm)} \Gamma_{\hat{i}\hat{j}}, & \{Q_+^{(+)}, Q_+^{(+)}\} &= 2i\mathcal{C}\Gamma_- \mathcal{P}_+ P_+, \\
 \{Q_+^{(-)}, Q_+^{(-)}\} &= 2i\mathcal{C}\Gamma_+ \mathcal{P}_- P_+ - i\frac{\lambda}{\sqrt{2}} \mathcal{C}\hat{\Gamma}^{\hat{i}\hat{j}} i\sigma_2 \ell_- \mathcal{P}_+ J_{\hat{i}\hat{j}} - i\frac{\lambda}{\sqrt{2}} \mathcal{C}\hat{\Gamma}^{\hat{i}\hat{j}} i\sigma_2 \ell_- \mathcal{P}_+ J_{\hat{i}\hat{j}}, \\
 \{Q_{\pm}^{(-)}, Q_{\mp}^{(-)}\} &= -2i\frac{\lambda}{\sqrt{2}} \mathcal{C}\hat{\Gamma}^{\hat{i}\hat{j}} i\sigma_2 \ell_- \mathcal{P}_{\mp} J_{\hat{i}\hat{j}}, \\
 \{Q_+^{(+)}, Q_+^{(-)}\} &= 2i\mathcal{C}\Gamma^{\hat{i}} \ell_- \mathcal{P}_+ P_{\hat{i}} + i\lambda \mathcal{C}\hat{\Gamma}^{\hat{i}} i\sigma_2 \ell_- \mathcal{P}_+ P_{\hat{i}}^*, \\
 \{Q_{\pm}^{(+)}, Q_{\mp}^{(-)}\} &= 2i\mathcal{C}\Gamma^{\hat{i}} \ell_- \mathcal{P}_{\mp} P_{\hat{i}} + i\lambda \mathcal{C}\hat{\Gamma}^{\hat{i}} i\sigma_2 \ell_- \mathcal{P}_{\mp} P_{\hat{i}}^*.
 \end{aligned} \quad (3.18)$$

In summary we have obtained the NH superalgebra of a pp-wave brane as (3.6), (3.7) and (3.18). This superalgebra can be derived from the NH superalgebra of an AdS brane (2.9) and (2.18) by an IW contraction.

We note that the NH superalgebra of a pp-wave brane contains a super-subalgebra

generated by $P_{\pm}, P_{\hat{i}}, P_{\hat{i}}^*, J_{\hat{i}\hat{j}}, J_{\hat{i}\hat{j}}$ and $Q_{\pm}^{(\pm)}$

$$\begin{aligned}
 [P_{-}, P_{\hat{i}}] &= -\frac{\lambda^2}{\sqrt{2}} P_{\hat{i}}^*, & [P_{-}, P_{\hat{i}}^*] &= \frac{1}{\sqrt{2}} P_{\hat{i}}, & [P_{\hat{i}}, P_{\hat{i}}^*] &= -\frac{1}{\sqrt{2}} \eta_{\hat{i}\hat{j}} P_{+}, \\
 [P_{\hat{i}}, J_{\hat{j}\hat{k}}] &= \eta_{\hat{i}\hat{j}} P_{\hat{k}} - \eta_{\hat{i}\hat{k}} P_{\hat{j}}, & [J_{\hat{i}\hat{j}}, P_{\hat{k}}^*] &= \eta_{\hat{j}\hat{k}} P_{\hat{i}}^* - \eta_{\hat{i}\hat{k}} P_{\hat{j}}^*, \\
 [J_{\hat{i}\hat{j}}, J_{\hat{k}\hat{l}}] &= \eta_{\hat{j}\hat{k}} J_{\hat{i}\hat{l}} + \text{3-terms}, & [J_{\hat{i}\hat{j}}, J_{\hat{k}\hat{l}}] &= \eta_{\hat{j}\hat{k}} J_{\hat{i}\hat{l}} + \text{3-terms}, \\
 [Q_{+}^{(-)}, P_{\hat{i}}] &= -\frac{1}{2\sqrt{2}} Q_{+}^{(+)} \Gamma_{\hat{i}} \Gamma_{+} f i \sigma_2, & [Q_{+}^{(+)}, P_{-}] &= -\frac{1}{\sqrt{2}} Q_{+}^{(+)} f i \sigma_2, \\
 [Q_{+}^{(-)}, P_{\hat{i}}^*] &= \frac{1}{2\sqrt{2}} Q_{+}^{(+)} \Gamma_{+} \Gamma_{\hat{i}}, & [Q_{+}^{(\bullet)}, J_{\hat{i}\hat{j}}] &= -\frac{1}{2} Q_{+}^{(\bullet)} \Gamma_{\hat{i}\hat{j}}, & [Q_{+}^{(\bullet)}, J_{\hat{i}\hat{j}}] &= -\frac{1}{2} Q_{+}^{(\bullet)} \Gamma_{\hat{i}\hat{j}}, \\
 \{Q_{+}^{(+)}, Q_{+}^{(+)}\} &= 2i\mathcal{C}\Gamma_{-} \mathcal{P}_{+} P_{+}, \\
 \{Q_{+}^{(-)}, Q_{+}^{(-)}\} &= 2i\mathcal{C}\Gamma_{+} \mathcal{P}_{-} P_{+} - i\frac{\lambda}{\sqrt{2}} \mathcal{C}\hat{\Gamma}^{\hat{i}\hat{j}} i\sigma_2 \ell_{-} \mathcal{P}_{+} J_{\hat{i}\hat{j}} - i\frac{\lambda}{\sqrt{2}} \mathcal{C}\hat{\Gamma}^{\hat{i}\hat{j}} i\sigma_2 \ell_{-} \mathcal{P}_{+} J_{\hat{i}\hat{j}}, \\
 \{Q_{+}^{(+)}, Q_{+}^{(-)}\} &= 2i\mathcal{C}\Gamma^{\hat{i}} \ell_{-} \mathcal{P}_{+} P_{\hat{i}} + i\lambda \mathcal{C}\hat{\Gamma}^{\hat{i}} i\sigma_2 \ell_{-} \mathcal{P}_{+} P_{\hat{i}}^*. \tag{3.19}
 \end{aligned}$$

This is regarded as a supersymmetrization of the pp-wave algebra which is the isometry on the brane worldvolume and the Lorentz symmetry in the transverse space. The existence of this super-subalgebra is ensured by the conditions (3.11) and (3.12).

3.2 Euclidean branes

We consider the case that $(+, -)$ are contained in the Dirichlet direction. Let us denote Neumann and Dirichlet directions as $\bar{A} = \hat{i}$ and $\underline{A} = (+, -, \hat{i})$, respectively. We derive the NH superalgebra of a Euclidean pp-wave brane as an IW contraction of the pp-wave superalgebra.

First we consider the bosonic subalgebra. We rescale generators in the pp-wave algebra as

$$P_{\underline{A}} \rightarrow \frac{1}{\Omega} P_{\underline{A}}, \quad J_{\hat{i}\hat{j}} \rightarrow \frac{1}{\Omega} J_{\hat{i}\hat{j}}, \quad P_{\hat{i}}^* \rightarrow \frac{1}{\Omega} P_{\hat{i}}^*, \tag{3.20}$$

and then take the limit $\Omega \rightarrow 0$. Under the contraction, we obtain the NH algebra of a Euclidean pp-wave brane

$$\begin{aligned}
 [P_{-}, P_{\hat{i}}] &= -\frac{\lambda^2}{\sqrt{2}} P_{\hat{i}}^*, & [P_{-}, P_{\hat{i}}^*] &= \frac{1}{\sqrt{2}} P_{\hat{i}}, & [P_{\hat{i}}, J_{\hat{j}\hat{k}}] &= \eta_{\hat{i}\hat{j}} P_{\hat{k}}, & [J_{\hat{i}\hat{j}}, P_{\hat{k}}^*] &= \eta_{\hat{j}\hat{k}} P_{\hat{i}}^*, \\
 [P_{\hat{i}}, P_{\hat{j}}^*] &= -\frac{1}{\sqrt{2}} \eta_{\hat{i}\hat{j}} P_{+}, & [P_{\hat{i}}, P_{\hat{j}}^*] &= -\frac{1}{\sqrt{2}} \eta_{\hat{i}\hat{j}} P_{+}, \tag{3.21}
 \end{aligned}$$

and (3.7).

To contract the fermionic part of the pp-wave superalgebra, we introduce a matrix

$$M = \ell \Gamma^{\bar{A}_1 \dots \bar{A}_p} \rho, \quad \ell^2 (-1)^{[\frac{p+1}{2}]} \rho^2 = 1 \tag{3.22}$$

and decompose $Q^{(\pm)}$ as

$$Q_{\pm}^{(\bullet)} = \pm Q_{\pm}^{(\bullet)} M \tag{3.23}$$

ρ	1-brane	3-brane	5-brane	7-brane	9-brane
σ_1, σ_3	(0,2),(2,0)		(2,4),(4,2)		-
$i\sigma_2$		(1,3),(3,1)		-	
1	(1,1)		(1,3),(3,1)		(5,5)

Table 3: Euclidean pp-wave branes

where $p = \text{odd}$ for the chirality of $Q^{(\pm)}$. We demand that (3.11) and (3.12) are satisfied. The first condition (3.11) is satisfied when

$$\pm(-1)^{p+\lfloor \frac{p+1}{2} \rfloor} = 1, \quad \rho^T = \pm\rho \quad (3.24)$$

so that $\rho^T = \rho$ for $p = 1 \pmod 4$ and $\rho^T = -\rho$ for $p = 3 \pmod 4$. It follows that $M' = -M$ and $\ell = \sqrt{-1}$. Next, the second condition (3.12) is found to be satisfied when

$$\pm(-1)^{p+n} = 1, \quad \rho = \begin{cases} 1, i\sigma_2 \\ \sigma_1, \sigma_3 \end{cases}. \quad (3.25)$$

This restricts the brane configuration as follows: (odd,odd)-branes with $\rho = 1$ and (even, even)-branes with $\rho = \sigma_1, \sigma_3$ for $p = 1 \pmod 4$, and (odd,odd)-branes with $\rho = i\sigma_2$ for $p = 3 \pmod 4$. We summarize the result in table 3. This shows possible 1/2 supersymmetric subspaces. Among them, (even,even)-branes of $p = 1 \pmod 4$ and (odd,odd)-branes of $p = 3 \pmod 4$ are 1/2 BPS D-branes of an open pp-wave superstring [30, 16].

Scaling $Q_{\pm}^{(\bullet)}$ as (3.17) and taking the limit $\Omega \rightarrow 0$, we obtain the fermionic part of the NH superalgebra of a Euclidean pp-wave brane

$$\begin{aligned}
 [Q_{\pm}^{(-)}, P_{\hat{i}}] &= -\frac{1}{2\sqrt{2}}Q_{\pm}^{(+)}\Gamma_{\hat{i}}\Gamma_+fi\sigma_2, & [Q_+^{(-)}, P_{\hat{i}}] &= -\frac{1}{2\sqrt{2}}Q_{\mp}^{(+)}\Gamma_{\hat{i}}\Gamma_+fi\sigma_2, \\
 [Q_+^{(+)}, P_-] &= -\frac{1}{\sqrt{2}}Q_-^{(+)}fi\sigma_2, & [Q_+^{(-)}, P_{\hat{i}}^*] &= \frac{1}{2\sqrt{2}}Q_-^{(+)}\Gamma_+\Gamma_{\hat{i}}, \\
 [Q_{\pm}^{(-)}, P_{\hat{i}}^*] &= \frac{1}{2\sqrt{2}}Q_{\pm}^{(+)}\Gamma_+\Gamma_{\hat{i}}, & [Q_{\pm}^{(\bullet)}, J_{\hat{i}\hat{j}}^*] &= -\frac{1}{2}Q_{\pm}^{(\bullet)}\Gamma_{\hat{i}\hat{j}}, & [Q_{\pm}^{(\bullet)}, J_{\hat{i}\hat{j}}] &= -\frac{1}{2}Q_{\pm}^{(\bullet)}\Gamma_{\hat{i}\hat{j}}, \\
 [Q_+^{(\bullet)}, J_{\hat{i}\hat{j}}] &= -\frac{1}{2}Q_-^{(\bullet)}\Gamma_{\hat{i}\hat{j}}, \\
 \{Q_{\pm}^{(+)}, Q_{\mp}^{(+)}\} &= 2i\mathcal{C}\Gamma_-P_+, \\
 \{Q_{\pm}^{(-)}, Q_{\mp}^{(-)}\} &= 2i\mathcal{C}\Gamma_+P_- - 2i\frac{\lambda}{\sqrt{2}}\mathcal{C}\hat{\Gamma}^{\hat{i}\hat{j}}i\sigma_2J_{\hat{i}\hat{j}}, \\
 \{Q_+^{(-)}, Q_+^{(-)}\} &= -i\frac{\lambda}{\sqrt{2}}\mathcal{C}\hat{\Gamma}^{\hat{i}\hat{j}}i\sigma_2J_{\hat{i}\hat{j}}^* - i\frac{\lambda}{\sqrt{2}}\mathcal{C}\hat{\Gamma}^{\hat{i}\hat{j}}i\sigma_2J_{\hat{i}\hat{j}}, \\
 \{Q_+^{(\pm)}, Q_+^{(\mp)}\} &= 2i\mathcal{C}\Gamma_{\mp}^{\hat{i}}\ell_{\mp}P_{\hat{i}} + i\lambda\mathcal{C}\hat{\Gamma}^{\hat{i}}\ell_{\mp}i\sigma_2P_{\hat{i}}^*, \\
 \{Q_+^{(\pm)}, Q_-^{(\mp)}\} &= 2i\mathcal{C}\Gamma_{\mp}^{\hat{i}}\ell_{\mp}P_{\hat{i}} + i\lambda\mathcal{C}\hat{\Gamma}^{\hat{i}}\ell_{\mp}i\sigma_2P_{\hat{i}}^*. \quad (3.26)
 \end{aligned}$$

Summarizing we have obtained the NH superalgebra of a Euclidean pp-wave brane as (3.21), (3.7) and (3.26). Obviously, this superalgebra can be derived from the NH superalgebra of an AdS brane (2.9) and (2.18) by an IW contraction.

We note that the NH superalgebra contains a super-subalgebra generated by $P_{\hat{i}}, P_{\hat{i}}^*, J_{\hat{i}\hat{j}}, J_{\hat{i}\hat{j}}^*$ and $Q_+^{(\pm)}$

$$\begin{aligned}
 [P_{\hat{i}}, J_{\hat{j}\hat{k}}] &= \eta_{\hat{i}\hat{j}} P_{\hat{k}} - \eta_{\hat{i}\hat{k}} P_{\hat{j}}, & [J_{\hat{i}\hat{j}}, P_{\hat{k}}^*] &= -\eta_{\hat{i}\hat{k}} P_{\hat{j}}^* + \eta_{\hat{j}\hat{k}} P_{\hat{i}}^*, \\
 [J_{\hat{i}\hat{j}}, J_{\hat{k}\hat{l}}] &= \eta_{\hat{j}\hat{k}} J_{\hat{i}\hat{l}} + \text{3-terms}, & [J_{\hat{i}\hat{j}}, J_{\hat{k}\hat{l}}^*] &= \eta_{\hat{j}\hat{k}} J_{\hat{i}\hat{l}}^* + \text{3-terms} \\
 [Q_+^{(-)}, P_{\hat{i}}] &= -\frac{1}{2\sqrt{2}} Q_+^{(+)} \Gamma_{\hat{i}} \Gamma_+ f i \sigma_2, & [Q_+^{(-)}, P_{\hat{i}}^*] &= \frac{1}{2\sqrt{2}} Q_+^{(+)} \Gamma_+ \Gamma_{\hat{i}}, \\
 [Q_+^{(\bullet)}, J_{\hat{i}\hat{j}}] &= -\frac{1}{2} Q_+^{(\bullet)} \Gamma_{\hat{i}\hat{j}}, & [Q_+^{(\bullet)}, J_{\hat{i}\hat{j}}^*] &= -\frac{1}{2} Q_+^{(\bullet)} \Gamma_{\hat{i}\hat{j}}, \\
 \{Q_+^{(-)}, Q_+^{(-)}\} &= -i \frac{\lambda}{\sqrt{2}} \mathcal{C} \hat{\Gamma}^{\hat{i}\hat{j}} i \sigma_2 J_{\hat{i}\hat{j}} - i \frac{\lambda}{\sqrt{2}} \mathcal{C} \hat{\Gamma}^{\hat{i}\hat{j}} i \sigma_2 J_{\hat{i}\hat{j}}^*, \\
 \{Q_+^{(\pm)}, Q_+^{(\mp)}\} &= 2i \mathcal{C} \Gamma^{\hat{i}} \ell_{\mp} P_{\hat{i}} + i \lambda \mathcal{C} \hat{\Gamma}^{\hat{i}} \ell_{\mp} i \sigma_2 P_{\hat{i}}^*
 \end{aligned} \tag{3.27}$$

which is regarded as a supersymmetrization of the Poincaré algebra generated by $\{P_{\hat{i}}, J_{\hat{i}\hat{j}}\}$ which is the isometry on the brane worldvolume and the Lorentz symmetry in the transverse space generated by $\{P_{\hat{i}}^*, J_{\hat{i}\hat{j}}^*\}$. The conditions (3.11) and (3.12) ensure the existence of this super-subalgebra.

4. Branes in $\text{AdS}_5 \times \text{S}^5$

A D-brane action [36] (see [37] for flat D-branes) is composed of the Dirac-Born-Infeld (DBI) action and the WZ action

$$S = S_{\text{DBI}} + S_{\text{WZ}}. \tag{4.1}$$

The DBI action is given, suppressing the dilaton and axion factors here, as

$$S_{\text{DBI}} = T \int_{\Sigma} \mathcal{L}_{\text{DBI}}, \quad \mathcal{L}_{\text{DBI}} = \sqrt{s \det(g + \mathcal{F})} d^{p+1} \xi \tag{4.2}$$

where $\mathcal{F} = F - \mathcal{B}$ and $F = dA$, and $s = -1$ for a Lorentzian brane while $s = 1$ for a Euclidean brane. g_{ij} is given by $g_{ij} = \mathbf{L}_i^A \mathbf{L}_j^B \eta_{AB}$ with $\mathbf{L}_i^A = \partial_i Z^M \mathbf{L}_M^A$. T is the tension of the brane. \mathcal{B} is the pullback of the NS-NS two-form and A is the gauge field on the worldvolume. For an F-string, the DBI action is replaced by the Nambu-Goto (NG) action

$$S_{\text{NG}} = T \int_{\Sigma} \mathcal{L}_{\text{NG}}, \quad \mathcal{L}_{\text{NG}} = \sqrt{s \det g} d^2 \xi. \tag{4.3}$$

The WZ action² is characterized by supersymmetric closed $(p+2)$ -form h_{p+2}

$$S_{\text{WZ}} = T \int_{\Sigma} \mathcal{L}_{\text{WZ}}, \quad h_{p+2} = d\mathcal{L}_{\text{WZ}} = \sum_{n=0} \frac{1}{n!} h^{(p+2-2n)} \mathcal{F}^n. \tag{4.4}$$

The closedness of h_{p+2}

$$0 = dh_{p+2} = \sum_{n=0} \frac{1}{n!} \left(dh^{(p+2-2n)} - h^{(p-2n)} d\mathcal{F} \right) \mathcal{F}^n \tag{4.5}$$

implies

$$dh^{(p+2-2n)} - h^{(p-2n)} d\mathcal{F} = 0. \tag{4.6}$$

²See [38] for the Roiban-Siegel formulation [39] of AdS D-branes.

4.1 CE-cohomology classification

In [20], it is shown that the Wess-Zumino (WZ) terms of p -branes in flat spacetime can be classified as non-trivial elements of the Chevalley-Eilenberg (CE) cohomology [21]. Let $C^p(\mathfrak{g}, \mathbb{R})$ be the vector space of p -cochains of a Lie algebra \mathfrak{g} . A p -cochain is a linear antisymmetric map: $\mathfrak{g} \times \cdots \times \mathfrak{g} \mapsto \mathbb{R}$ and a coboundary operator δ with $\delta^2 = 0$ acts as $C^p(\mathfrak{g}, \mathbb{R}) \mapsto C^{p+1}(\mathfrak{g}, \mathbb{R})$. The CE cohomology group $H^p(\mathfrak{g}, \mathbb{R})$ is defined by Z^p/B^p where Z^p and B^p are the vector spaces of p -cocycles $c \in Z^p$ satisfying $\delta c = 0$ and p -coboundaries $c \in B^p$ satisfying $c = \delta c'$ with $c' \in C^{p-1}(\mathfrak{g}, \mathbb{R})$, respectively. In the present context, this is viewed as the de Rham cohomology group $E^p(G, \mathbb{R})$ for left-invariant (LI) p -forms on the supergroup $G = \text{“super-Poincaré”}/\text{“Lorentz”}$, for which a non-trivial element of the cohomology is a closed LI p -form modulo exact LI p -forms on G . This is generalized to D-branes in [22, 23] by introducing an additional two form which corresponds to the modified field strength of background \mathcal{B} field.

Here we examine WZ terms of AdS branes by using the CE cohomology on \mathfrak{g} of the supergroup $G = \text{PSU}(2,2|4)/(\text{SO}(4,1) \times \text{SO}(5))$, i.e. “super-AdS₅ × S⁵”/“Lorentz”. We show that except for the $p = 1$ case h_{p+2} can be obtained as a Lorentz invariant non-trivial element of the CE-cohomology on the free differential algebra which is the MC equations (2.6) corresponding to the super-AdS₅ × S⁵ algebra (2.1) equipped with

$$d\mathcal{F} = -i\mathbf{L}^A \bar{L} \Gamma_A \sigma L \tag{4.7}$$

where σ is σ_3 for D-branes while $-\sigma_1$ for F1- and NS5-branes.

In order not to introduce an additional dimensionful parameter we assign a dimension to Cartan one-forms as follows

$$\begin{array}{ccccccc} \mathbf{L}^A & L^\alpha & \mathbf{L}^{AB} & \lambda & \mathcal{F} & h_{p+2} & h^{(k)} \\ \dim & 1 & 1/2 & 0 & -1 & 2 & p+1 & k-1 \end{array} \tag{4.8}$$

where $\dim h_{p+2} = p + 1$ because $\dim h_{p+2} = \dim \mathcal{L}_{\text{WZ}}^p = \dim \mathcal{L}_{BI}^p = p + 1$ for structureless fundamental branes.

Suppose that $h^{(k)}$ is of the form $(\mathbf{L}^A)^n (L^\alpha)^m \lambda^l$, then n , m and l must satisfy

$$n + \frac{1}{2}m - l = k - 1, \quad n + m = k, \tag{4.9}$$

because $h^{(k)}$ is a Lorentz invariant k -form of dimension $k - 1$. We require that $\epsilon_{a_1 \dots a_5}$ and $\epsilon_{a'_1 \dots a'_5}$ are accompanied with λ ; $\lambda \epsilon_{a_1 \dots a_5}$ and $\lambda \epsilon_{a'_1 \dots a'_5}$, because $\epsilon_{a_1 \dots a_5}$ and $\epsilon_{a'_1 \dots a'_5}$ disappear in the flat limit $\lambda \rightarrow 0$. Requiring $l \geq 0$ because otherwise $h^{(k)}$ diverges in the flat limit. This implies $l = -\frac{1}{2}m + 1 \leq 1$ and so we consider $l = 0, 1$. Since (4.9) is satisfied for $(m, n) = (2, k - 2), (0, k)$ for $l = 0, 1$, respectively, we find that $h^{(k)}$, $k = 1, 3, 5, \dots$, has

the following form

$$h^{(1)} = 0, \quad (4.10)$$

$$h^{(3)} = c_0^{(3)} \mathbf{L}^a \bar{\Gamma}_a \varrho_0^{(3)} L + c_1^{(3)} \mathbf{L}^{a'} \bar{\Gamma}_{a'} \varrho_1^{(3)} L, \quad (4.11)$$

$$h^{(5)} = c_0^{(5)} \mathbf{L}^{a_1} \mathbf{L}^{a_2} \mathbf{L}^{a_3} \bar{\Gamma}_{a_1 a_2 a_3} \varrho_0^{(5)} L + \dots + c_3^{(5)} \mathbf{L}^{a'_1} \mathbf{L}^{a'_2} \mathbf{L}^{a'_3} \bar{\Gamma}_{a'_1 a'_2 a'_3} \varrho_3^{(5)} L \\ + b_0 \lambda \epsilon_{a_1 \dots a_5} \mathbf{L}^{a_1} \dots \mathbf{L}^{a_5} + b_5 \lambda \epsilon_{a'_1 \dots a'_5} \mathbf{L}^{a'_1} \dots \mathbf{L}^{a'_5}, \quad (4.12)$$

$$h^{(7)} = c_0^{(7)} \mathbf{L}^{a_1} \dots \mathbf{L}^{a_5} \bar{\Gamma}_{a_1 \dots a_5} \varrho_0^{(7)} L + \dots + c_5^{(7)} \mathbf{L}^{a'_1} \dots \mathbf{L}^{a'_5} \bar{\Gamma}_{a'_1 \dots a'_5} \varrho_5^{(7)} L, \quad (4.13)$$

$$h^{(9)} = c_0^{(9)} \mathbf{L}^{a_1} \dots \mathbf{L}^{a_7} \bar{\Gamma}_{a_1 \dots a_7} \varrho_0^{(9)} L + \dots + c_7^{(9)} \mathbf{L}^{a'_1} \dots \mathbf{L}^{a'_7} \bar{\Gamma}_{a'_1 \dots a'_7} \varrho_7^{(9)} L, \quad (4.14)$$

where $c_i^{(k)}$ and b_i are constants determined below. $\varrho_i^{(k)}$ are 2×2 matrices satisfying $\varrho^{(k)T} = \varrho^{(k)}$ for $k = 3, 7$ while $\varrho^{(k)T} = -\varrho^{(k)}$ for $k = 5, 9$, because $\mathcal{C}\Gamma^{A_1 \dots A_N}$ is symmetric for $N = 1, 2 \pmod 4$ and anti-symmetric otherwise.

It is straightforward to solve (4.6) to determine coefficients and $\varrho_i^{(k)}$. We find

$$h^{(1)} = 0, \quad (4.15)$$

$$h^{(3)} = c \mathbf{L}^A \bar{\Gamma}_A \varrho L, \quad (4.16)$$

$$h^{(5)} = \frac{c}{3!} \left[\mathbf{L}^{A_1} \mathbf{L}^{A_2} \mathbf{L}^{A_3} \bar{\Gamma}_{A_1 A_2 A_3} i \sigma_2 L + \frac{i}{5} \lambda (\epsilon_{a_1 \dots a_5} \mathbf{L}^{a_1} \dots \mathbf{L}^{a_5} - \epsilon_{a'_1 \dots a'_5} \mathbf{L}^{a'_1} \dots \mathbf{L}^{a'_5}) \right], \quad (4.17)$$

$$h^{(7)} = \frac{c}{5!} \mathbf{L}^{A_1} \dots \mathbf{L}^{A_5} \bar{\Gamma}_{A_1 \dots A_5} \varrho L, \quad (4.18)$$

$$h^{(9)} = \frac{c}{7!} \mathbf{L}^{A_1} \dots \mathbf{L}^{A_7} \bar{\Gamma}_{A_1 \dots A_7} i \sigma_2 L. \quad (4.19)$$

In appendix B, $c = c_0^{(3)}$ is determined by the κ -invariance [36] of the total action S as $c = i$ and 1 for Lorentzian and Euclidean branes respectively: $c = \sqrt{s}$. ϱ is $\sigma_1(\sigma_3)$ for $\sigma = \sigma_3(-\sigma_1)$ respectively. The closedness (4.6) is ensured by the Fierz identities

$$(\mathcal{C}\Gamma_A)_{(\alpha\beta} (\mathcal{C}\Gamma^A \varrho)_{\gamma\delta)} = 0, \\ (\mathcal{C}\Gamma_C)_{(\alpha\beta} (\mathcal{C}\Gamma^{ABC} i \sigma_2)_{\gamma\delta)} + 2(\mathcal{C}\Gamma^{[A} \varrho)_{(\alpha\beta} (\mathcal{C}\Gamma^{B]} \sigma)_{\gamma\delta)} = 0, \\ (\mathcal{C}\Gamma_B)_{(\alpha\beta} (\mathcal{C}\Gamma^{A_1 \dots A_4 B} \varrho)_{\gamma\delta)} + 4(\mathcal{C}\Gamma^{[A_1 A_2 A_3} i \sigma_2)_{(\alpha\beta} (\mathcal{C}\Gamma^{A_4]} \sigma)_{\gamma\delta)} = 0, \\ (\mathcal{C}\Gamma_B)_{(\alpha\beta} (\mathcal{C}\Gamma^{A_1 \dots A_6 B} i \sigma_2)_{\gamma\delta)} + 6(\mathcal{C}\Gamma^{[A_1 \dots A_5} \varrho)_{(\alpha\beta} (\mathcal{C}\Gamma^{A_6]} \sigma)_{\gamma\delta)} = 0. \quad (4.20)$$

In summary, closed $(p+2)$ -forms h_{p+2} are composed in terms of $h^{(k)}$ found above as in (4.4). The actions S for F1- and D3-branes coincide with those obtained in [40] and [41], respectively.

We show that h_{p+2} is a non-trivial element of the cohomology except for h_3 . If h_{p+2} is exact, there exists b_{p+1} such as $h_{p+2} = db_{p+1}$. Since

$$h_3 = db_2, \quad b_2 = -c \lambda^{-1} \bar{\mathcal{L}} \mathcal{I} \varrho i \sigma_2 L \quad (4.21)$$

h_3 is a trivial element of the cohomology [24, 25]. Next we show that h_{p+2} with $p = 3, 5, 7$ is not exact. Let us examine a term of the form $\frac{1}{(\frac{p-1}{2})!} h^{(3)} \mathcal{F}^{\frac{p-1}{2}}$ contained in h_{p+2} . We note

that \mathcal{F} can be written as³

$$\mathcal{F} = ic\lambda^{-1}\bar{L}\mathcal{I}\sigma i\sigma_2 L \quad (4.22)$$

up to an exact form, and that there does not exist a one-form supercurrent f such that $\mathcal{F} = df$. So b_{p+1} must contain a term of the form $\bar{L}\mathcal{I}\rho i\sigma_2 L \mathcal{F}^{\frac{p-1}{2}}$. Differentiating it, we have $\bar{L}\mathcal{I}\rho i\sigma_2 L L^A \bar{L}\Gamma_A \sigma L \mathcal{F}^{\frac{p-3}{2}}$ in addition to $\frac{1}{(\frac{p-1}{2})!} h^{(3)} \mathcal{F}^{\frac{p-1}{2}}$. For h_{p+2} to be exact, this term must be canceled by the differential of a term which is a $(p+1)$ -form with $p-1$ L^α 's. From the MC equation (2.6), we see that there does not exist such a term. Thus h_{p+2} with $p = 3, 5, 7$ obtained above are non-trivial elements of the cohomology.

4.2 $(p+1)$ -dimensional form of the WZ term

In this subsection, we give the $(p+1)$ -dimensional form of the WZ term h_{p+2} . We follow [40, 41] in which the $(p+1)$ -dimensional form of the WZ term of F1- and D3-branes are given.

The LI Cartan one-forms satisfy the following differential equations

$$\partial_t \hat{\mathbf{L}}^A = 2i\bar{\theta}\Gamma^A \hat{L}, \quad (4.23)$$

$$\partial_t \hat{L}^\alpha = d\theta + \frac{\lambda}{2} \hat{\mathbf{L}}^A \hat{\Gamma}_A i\sigma_2 \theta + \frac{1}{4} \hat{\mathbf{L}}^{AB} \Gamma_{AB} \theta, \quad (4.24)$$

$$\partial_t \hat{\mathbf{L}}^{AB} = -2i\lambda \bar{\theta} \hat{\Gamma}^{AB} i\sigma_2 \hat{L} \quad (4.25)$$

where a ‘‘hat’’ on a supercurrent implies that θ is rescaled as $\theta \rightarrow t\theta$. First we note that

$$\partial_t d\hat{\mathcal{F}} = -\partial_t d\hat{\mathcal{B}} = -2id(\hat{\mathbf{L}}^A \hat{L}\Gamma_A \sigma \theta). \quad (4.26)$$

This is solved by

$$\mathcal{B} = 2i \int_0^1 dt \hat{\mathbf{L}}^A \hat{L}\Gamma_A \sigma \theta + B^{(2)}. \quad (4.27)$$

where $B^{(2)}$ is a bosonic 2-form satisfying $dB^{(2)} = 0$. Thus we obtain

$$\mathcal{F} = F - 2i \int_0^1 dt \hat{\mathbf{L}}^A \hat{L}\Gamma_A \sigma \theta - B^{(2)}, \quad (4.28)$$

$$\partial_t \hat{\mathcal{F}} = -2i(\hat{\mathbf{L}}^A \hat{L}\Gamma_A \sigma \theta). \quad (4.29)$$

For D-brane actions, we choose $B^{(2)} = 0$.

By using (4.23)-(4.25) and (4.29), one sees that the closed $(p+1)$ -form h_{p+2} satisfies

$$\partial_t \hat{h}_{p+2} = db_{p+1}, \quad (4.30)$$

where

$$b_{p+1} = [C \wedge e^{\hat{\mathcal{F}}}]_{p+1}, \quad C = \bigoplus_{\ell=\text{even}} C^{(\ell)},$$

$$C^{(2n)} = \frac{2\sqrt{s}}{(2n-1)!} \hat{\mathbf{L}}^{A_1} \dots \hat{\mathbf{L}}^{A_{2n-1}} \hat{L}\Gamma_{A_1 \dots A_{2n-1}} (\sigma)^n i\sigma_2 \theta. \quad (4.31)$$

³This implies that $h_2 = h^{(2)} + h^{(0)} \mathcal{F}$ with $h^{(0)} = i\hat{c}\lambda$ and $h^{(2)} = -\hat{c}\bar{L}\mathcal{I}\sigma i\sigma_2 L$ can be a nontrivial element of the cohomology. It is interesting to examine the 0-brane action with the WZ term h_2 .

It follows that

$$\int_B h_{p+2} = \int_\Sigma \mathcal{L}_{\text{WZ}} = \int_\Sigma \left[\int_0^1 dt b_{p+1} + C^{(p+1)} \right] \quad (4.32)$$

where $\partial B = \Sigma$, and $C^{(p+1)}$ is a bosonic $(p+1)$ -form satisfying

$$h_{p+2}|_{\text{bosonic}} = dC^{(p+1)}. \quad (4.33)$$

Letting $p = 1$ and $\sigma = -\sigma_1$, we reproduce the WZ term of an F-string.

5. Non-relativistic branes in $\text{AdS}_5 \times \text{S}^5$

In [15], the non-relativistic F-string in $\text{AdS}_5 \times \text{S}^5$ is examined. There the leading contributions of the NG and the WZ parts in the non-relativistic limit cancel each other, and the next-to-leading terms contribute to the non-relativistic F-string action. Thus, in order to extract non-relativistic brane actions, we need to know the next-to-leading order terms in the limit $\Omega \rightarrow 0$. Let us consider the scaling

$$X^A \rightarrow \Omega X^A, \quad \theta_- \rightarrow \Omega \theta_-, \quad (5.1)$$

$$T = \Omega^{-2} T_{\text{NR}}, \quad F = \Omega F_1. \quad (5.2)$$

(5.1) is consistent with the scaling (2.8) and (2.17). It is straightforward to see that by substituting (5.1) into the concrete expression of the supercurrents given in appendix A.1, \mathbf{L}^A and L are expanded as

$$\begin{aligned} \mathbf{L}^{\bar{A}} &= \sum_{n=0} \Omega^{2n} \mathbf{L}_{2n}^{\bar{A}}, & \mathbf{L}^A &= \sum_{n=0} \Omega^{2n+1} \mathbf{L}_{2n+1}^A, \\ L_+ &= \sum_{n=0} \Omega^{2n} L_{+2n}, & L_- &= \sum_{n=0} \Omega^{2n+1} L_{-2n+1}. \end{aligned} \quad (5.3)$$

Expand \mathbf{L}^{AB} as

$$\mathbf{L}^{\bar{A}\bar{B}} = \sum_{n=0} \Omega^{2n} \mathbf{L}_{2n}^{\bar{A}\bar{B}}, \quad \mathbf{L}^{AB} = \sum_{n=0} \Omega^{2n} \mathbf{L}_{2n}^{AB}, \quad \mathbf{L}^{\bar{A}B} = \sum_{n=0} \Omega^{2n+1} \mathbf{L}_{2n+1}^{\bar{A}B}, \quad (5.4)$$

and substitute (5.3) and (5.4) into the MC equation (2.6) for the super- $\text{AdS}_5 \times \text{S}^5$ algebra, then the LI Cartan one-forms $\{\mathbf{L}_0^{\bar{A}}, \mathbf{L}_1^{\bar{A}}, L_{+0}, L_{-1}, \mathbf{L}_0^{\bar{A}\bar{B}}, \mathbf{L}_0^{AB}, \mathbf{L}_1^{\bar{A}B}\}$ form the MC equations (2.20)-(2.28) for the NH superalgebra.⁴

We consider the non-relativistic limit of the AdS branes obtained in the previous section. In the following subsections, we will show that when we introduce

$$M = \sqrt{-s} \Gamma^{\bar{A}_0 \dots \bar{A}_p} \otimes \rho \quad (5.5)$$

⁴As will be seen below, the non-relativistic actions are composed of $\{\mathbf{L}_0^{\bar{A}}, \mathbf{L}_2^{\bar{A}}, \mathbf{L}_1^{\bar{A}}, L_{+0}, L_{+2}, L_{-1}\}$. So these actions are not invariant under the NH superalgebra, but under an expanded superalgebra [24, 43] (see also [44, 45]) which is a generalization of the IW contraction [4], generated by generators dual to $\{\mathbf{L}_m^{\bar{A}}, \mathbf{L}_m^{\bar{A}}, L_{\pm m}, \mathbf{L}_m^{\bar{A}\bar{B}}, \mathbf{L}_m^{AB}, \mathbf{L}_m^{\bar{A}B} \mid 0 \leq m \leq 2\}$.

with $\rho = \sigma_1(i\sigma_2)$ for D p -branes with $p = 1(3) \bmod 4$, respectively, and with $\rho = \sigma_3$ for F1 and NS5, AdS p -brane actions admit expansion

$$S = T_{\text{NR}} \int_{\Sigma} \left[\Omega^{-2} (\mathcal{L}_{\{\text{DBI}\}}^{\text{div}_{\text{NG}}} + \mathcal{L}_{\text{WZ}}^{\text{div}}) + \mathcal{L}_{\{\text{DBI}\}}^{\text{fin}_{\text{NG}}} + \mathcal{L}_{\text{WZ}}^{\text{fin}} + O(\Omega^2) \right]. \quad (5.6)$$

For the consistent non-relativistic limit $\Omega \rightarrow 0$, the divergent term $\int (\mathcal{L}_{\{\text{DBI}\}}^{\text{div}_{\text{NG}}} + \mathcal{L}_{\text{WZ}}^{\text{div}})$ should cancel out. First, we show that

$$d\mathcal{L}_{\{\text{DBI}\}}^{\text{div}_{\text{NG}}} + h_{p+2}^{\text{div}} = 0. \quad (5.7)$$

This implies that the divergent terms with θ cancel out, since h_{p+2}^{div} is composed of only terms with θ . Next, we consider the bosonic terms of $\mathcal{L}_{\{\text{DBI}\}}^{\text{div}_{\text{NG}}} + \mathcal{L}_{\text{WZ}}^{\text{div}}$

$$\frac{1}{(p+1)!} \epsilon_{\bar{A}_0 \dots \bar{A}_p} e_0^{\bar{A}_0} \dots e_0^{\bar{A}_p} + C_0^{(p+1)} \quad (5.8)$$

where $C_0^{(p+1)}$ is the leading contribution of $C^{(p+1)}$ in (4.32). This is deleted by choosing $C_0^{(p+1)} = -\frac{1}{(p+1)!} \epsilon_{\bar{A}_0 \dots \bar{A}_p} e_0^{\bar{A}_0} \dots e_0^{\bar{A}_p}$. It is easy to see that $dC_0^{(p+1)} = 0$ by using the expressions given in appendix A.1. Thus the bosonic divergent terms also cancel out. As a result, we derive the non-relativistic brane action

$$S_{\text{NR}} = T_{\text{NR}} \int_{\Sigma} \mathcal{L}_{\text{NR}}, \quad \mathcal{L}_{\text{NR}} = \mathcal{L}_{\{\text{DBI}\}}^{\text{fin}_{\text{NG}}} + \mathcal{L}_{\text{WZ}}^{\text{fin}} \quad (5.9)$$

which is drastically simplified by gauge fixing the κ -symmetry by $\theta_+ = 0$. We examine each AdS branes in turn below.

5.1 F-string

First, we consider an F-string. The 3-form h_3 is given in (4.16) with $\varrho = \sigma_3$. The gluing matrix M is

$$M = \sqrt{-s} \Gamma^{\bar{A}_0 \bar{A}_1} \otimes \rho, \quad \rho = \sigma_1, \sigma_3, 1. \quad (5.10)$$

Since

$$M' \Gamma_{\bar{A}} \varrho = \Gamma_{\bar{A}} M \varrho = \pm \Gamma_{\bar{A}} \varrho M, \quad \rho = \begin{cases} \sigma_3, 1 \\ \sigma_1 \end{cases}, \quad (5.11)$$

h_3 is expanded as

$$Th_3 = T_{\text{NR}} \Omega^{-2} h_3^{\text{div}} + T_{\text{NR}} h_3^{\text{fin}} + O(\Omega^2), \quad (5.12)$$

$$h_3^{\text{div}} = \sqrt{s} \mathbf{L}_0^{\bar{A}} \bar{L}_{+0} \Gamma_{\bar{A}} \varrho L_{+0}, \quad (5.13)$$

$$h_3^{\text{fin}} = \sqrt{s} \left[\mathbf{L}_0^{\bar{A}} \bar{L}_{-1} \Gamma_{\bar{A}} \varrho L_{-1} + \mathbf{L}_2^{\bar{A}} \bar{L}_{+0} \Gamma_{\bar{A}} \varrho L_{+0} + 2\mathbf{L}_0^{\bar{A}} \bar{L}_{+0} \Gamma_{\bar{A}} \varrho L_{+2} + 2\mathbf{L}_1^{\bar{A}} \bar{L}_{+0} \Gamma_{\bar{A}} \varrho L_{-1} \right], \quad (5.14)$$

for $\rho = \sigma_3, 1$, while h_3 is of order Ω for $\rho = \sigma_1$. On the other hand, the NG part is expanded as

$$T\mathcal{L}_{\text{NG}} = T_{\text{NR}}\Omega^{-2}\mathcal{L}_{\text{NG}}^{\text{div}} + T_{\text{NR}}\mathcal{L}_{\text{NG}}^{\text{fn}} + O(\Omega^2), \quad (5.15)$$

$$\mathcal{L}_{\text{NG}}^{\text{div}} = \sqrt{s \det g_0} d^2 \xi = \det((\mathbf{L}_0^{\bar{A}})_i) d^2 \xi = \frac{1}{2} \epsilon_{\bar{A}\bar{B}} (\mathbf{L}_0^{\bar{A}})(\mathbf{L}_0^{\bar{B}}), \quad (5.16)$$

$$\mathcal{L}_{\text{NG}}^{\text{fn}} = \frac{1}{2} \sqrt{s \det g_0} g_0^{ij} (g_2)_{ij} d^2 \xi, \quad (5.17)$$

with $\epsilon_{\bar{A}_0 \bar{A}_1} = 1$ and

$$(g_0)_{ij} = (\mathbf{L}_0^{\bar{A}})_i (\mathbf{L}_0^{\bar{B}})_j \eta_{\bar{A}\bar{B}}, \quad (5.18)$$

$$(g_2)_{ij} = 2(\mathbf{L}_0^{\bar{A}})_i (\mathbf{L}_2^{\bar{B}})_j \eta_{\bar{A}\bar{B}} + (\mathbf{L}_1^{\underline{A}})_i (\mathbf{L}_1^{\underline{B}})_j \eta_{\underline{A}\underline{B}}. \quad (5.19)$$

The leading contribution satisfies [15]

$$d\mathcal{L}_{\text{NG}}^{\text{div}} = \epsilon_{\bar{A}\bar{B}} i \bar{L}_{+0} \Gamma^{\bar{A}} L_{+0} \mathbf{L}_0^{\bar{B}} = -\sqrt{s} \bar{\mathbf{L}}_0^{\bar{A}} \bar{L}_{+0} \Gamma_{\bar{A}} \rho L_{+0} \quad (5.20)$$

where we have used (2.20) and $L_+ = M L_+$. This cancels out h_3^{div} in (5.13) only when $\varrho = \rho$

$$d\mathcal{L}_{\text{NG}}^{\text{div}} + h_3^{\text{div}} = 0. \quad (5.21)$$

This implies that θ -dependent terms in $\mathcal{L}_{\text{NG}}^{\text{div}} + \mathcal{L}_{\text{WZ}}^{\text{div}}$ cancel each other. The bosonic term of $\mathcal{L}_{\text{NG}}^{\text{div}}$ in (5.16), $\frac{1}{2} \epsilon_{\bar{A}\bar{B}} e_0^{\bar{A}} e_0^{\bar{B}}$, is deleted by choosing $C_0^{(2)}$ in (4.32) as

$$C_0^{(2)} = -\frac{1}{2} \epsilon_{\bar{A}\bar{B}} e_0^{\bar{A}} e_0^{\bar{B}} \quad (5.22)$$

which satisfies $dC_0^{(2)} = 0$. Thus, the gluing matrix (5.10) with $\rho = \sigma_3$ leads to the consistent non-relativistic limit of the F-string. The non-relativistic F-string action is (5.9) with (5.17) and

$$\begin{aligned} \mathcal{L}_{\text{WZ}}^{\text{fn}} = & \int_0^1 dt 2\sqrt{s} \left[\hat{\mathbf{L}}_0^{\bar{A}} (\hat{L}_{-1} \Gamma_{\bar{A}} \varrho \theta_- + \hat{L}_{+2} \Gamma_{\bar{A}} \varrho \theta_+) \right. \\ & \left. + \hat{\mathbf{L}}_1^{\underline{A}} (\hat{L}_{-1} \Gamma_{\underline{A}} \varrho \theta_+ + \hat{L}_{+0} \Gamma_{\underline{A}} \varrho \theta_-) + \hat{\mathbf{L}}_2^{\bar{A}} \hat{L}_{+0} \Gamma_{\bar{A}} \varrho \theta_+ \right]. \end{aligned} \quad (5.23)$$

We fix the κ -gauge symmetry of the action by $\theta_+ = 0$ (see appendix B). Then we have

$$\begin{aligned} \mathbf{L}_0^{\bar{A}} &= e_0^{\bar{A}}, \quad \mathbf{L}_2^{\bar{A}} = e_2^{\bar{A}} + i\bar{\theta}_- \Gamma^{\bar{A}} D\theta_-, \quad \mathbf{L}_1^{\underline{A}} = e_1^{\underline{A}}, \\ L_{-1} &= D\theta_-, \quad D\theta_- \equiv d\theta_- + \frac{\lambda}{2} e_0^{\bar{A}} \hat{\Gamma}_{\bar{A}} i\sigma_2 \theta_- + \frac{1}{4} \omega_0^{\bar{A}\bar{B}} \Gamma_{\bar{A}\bar{B}} \theta_-, \\ (g_0)_{ij} &= (e_0^{\bar{A}})_i (e_0^{\bar{B}})_j \eta_{\bar{A}\bar{B}}. \end{aligned} \quad (5.24)$$

In the static gauge, $x^{\bar{A}} = \xi^i$, $(e_0^{\bar{A}})_i$ is the vielbein on the AdS brane worldvolume. Thanks to the κ -gauge fixing, we can perform the t -integration in (5.23) easily. $\mathcal{L}_{\text{NG}}^{\text{fn}}$ is reduced to

$$\mathcal{L}_{\text{NG}}^{\text{fn}} = d^2 \xi \sqrt{s \det g_0} \left[g_0^{ij} (e_0^{\bar{A}})_i (e_2^{\bar{B}})_j \eta_{\bar{A}\bar{B}} + \frac{1}{2} g_0^{ij} (e_1^{\underline{A}})_i (e_1^{\underline{B}})_j \eta_{\underline{A}\underline{B}} + i g_0^{ij} \bar{\theta}_- \gamma_i D_j \theta_- \right] \quad (5.25)$$

where $\gamma_i = (e_0^{\bar{A}})_i \Gamma_{\bar{A}}$. By parameterizing the group manifold as in appendix A.1, it is rewritten as

$$\mathcal{L}_{\text{NG}}^{\text{fin}} = d^2\xi \sqrt{s \det g_0} \left[\frac{1}{2} g_0^{ij} \partial_i y^A \partial_j y^B \eta_{AB} + \frac{\lambda^2}{2} (my^2 - ny'^2) + i\bar{\theta}_- \gamma^i D_i \theta_- \right] \quad (5.26)$$

for an (m, n) -brane with $(m, n) = (2, 0), (0, 2)$. On the other hand, $\mathcal{L}_{\text{WZ}}^{\text{fin}}$ is reduced to

$$\mathcal{L}_{\text{WZ}}^{\text{fin}} = \sqrt{s} e_0^{\bar{A}} D\bar{\theta}_- \Gamma_{\bar{A}} \varrho \theta_- = d^2\xi \sqrt{s \det g_0} \left[-i D_i \bar{\theta}_- \gamma^i \theta_- \right] \quad (5.27)$$

where we have used $\theta_- = -M\theta_-$ in the second equality. Combining these results, we obtain the non-relativistic action

$$S_{\text{NR}}^{F1} = T_{\text{NR}} \int d^2\xi \sqrt{s \det g_0} \left[\frac{1}{2} g_0^{ij} \partial_i y^A \partial_j y_A + \frac{\lambda^2}{2} (my^2 - ny'^2) + 2i\bar{\theta}_- \gamma^i D_i \theta_- \right]. \quad (5.28)$$

This is a free field action of scalars and fermions propagating on $(2,0)$ - or $(0,2)$ -brane worldvolume. For the case of a Lorentzian $(2,0)$ -brane, this reproduces the non-relativistic AdS₂ brane action obtained in [15].

5.2 D-string

Secondly, we consider a D-string, for which $\varrho = \sigma_1$ and $\sigma = \sigma_3$. The gluing matrix M is given in (5.10). Since

$$M' \Gamma_{\bar{A}} \varrho = \Gamma_{\bar{A}} M \varrho = \pm \Gamma_{\bar{A}} \varrho M, \quad \rho = \begin{cases} \sigma_1, 1 \\ \sigma_3 \end{cases}, \quad (5.29)$$

h_3 is expanded as (5.12) with $\varrho = \sigma_1$ for $\rho = \sigma_1, 1$, while h_3 is of order Ω for $\rho = \sigma_3$. We note that for $\rho = \sigma_1$, \mathcal{F} is of order Ω

$$\mathcal{F} = \Omega \mathcal{F}_1 + O(\Omega^3), \quad (5.30)$$

$$\mathcal{F}_1 = F_1 - 2i \int_0^1 dt \left[\hat{\mathbf{L}}_0^{\bar{A}} (\hat{L}_{+0} \Gamma_{\bar{A}} \sigma \theta_- + \hat{L}_{-1} \Gamma_{\bar{A}} \sigma \theta_+) + \hat{\mathbf{L}}_1^{\bar{A}} \hat{L}_{+0} \Gamma_{\bar{A}} \sigma \theta_+ \right] \quad (5.31)$$

since

$$M' \Gamma_{\bar{A}} \sigma = -\Gamma_{\bar{A}} \sigma M. \quad (5.32)$$

So, the DBI part is expanded as

$$T \mathcal{L}_{\text{DBI}} = T_{\text{NR}} \Omega^{-2} \mathcal{L}_{\text{DBI}}^{\text{div}} + T_{\text{NR}} \mathcal{L}_{\text{DBI}}^{\text{fin}} + O(\Omega^4), \quad (5.33)$$

$$\mathcal{L}_{\text{DBI}}^{\text{div}} = \sqrt{s \det g_0} d^2\xi, \quad (5.34)$$

$$\mathcal{L}_{\text{DBI}}^{\text{fin}} = \frac{1}{2} \sqrt{s \det g_0} \left(g_0^{ij} (g_2)_{ij} - \frac{1}{2} g_0^{ik} (\mathcal{F}_1)_{kj} g_0^{jl} (\mathcal{F}_1)_{li} \right) d^2\xi \quad (5.35)$$

where g_0 , g_2 and \mathcal{F}_1 are given in (5.18), (5.19) and (5.31), respectively. For $\rho = 1, \sigma_3$, \mathcal{F} is of order Ω^0 . As was done for the F-string case, the h_3^{div} in (5.13) with $\varrho = \sigma_1$ and the leading contribution of the fermionic part of the DBI action cancel each other. By choosing

$C_0^{(2)} = -\frac{1}{2}\epsilon_{\bar{A}\bar{B}}e_0^{\bar{A}}e_0^{\bar{B}}$, the bosonic terms of the divergent part cancel out. Thus, the gluing matrix with $\rho = \sigma_1$ leads to the consistent non-relativistic limit of the D-string.

The non-relativistic D-string action is given by (5.9) with (5.35) and

$$\begin{aligned} \mathcal{L}_{\text{WZ}}^{\text{fn}} = & \int_0^1 dt 2\sqrt{s} \left[\hat{\mathbf{L}}_0^{\bar{A}} (\hat{\bar{L}}_{-1} \Gamma_{\bar{A}} \varrho \theta_- + \hat{\bar{L}}_{+2} \Gamma_{\bar{A}} \varrho \theta_+) \right. \\ & \left. + \hat{\mathbf{L}}_1^{\underline{A}} (\hat{\bar{L}}_{-1} \Gamma_{\underline{A}} \varrho \theta_+ + \hat{\bar{L}}_{+0} \Gamma_{\underline{A}} \varrho \theta_-) + \hat{\mathbf{L}}_2^{\bar{A}} \hat{\bar{L}}_{+0} \Gamma_{\bar{A}} \varrho \theta_+ \right]. \end{aligned} \quad (5.36)$$

Let us gauge fix the κ -gauge symmetry by choosing $\theta_+ = 0$. This makes it easy to perform the t -integration in $\mathcal{L}_{\text{WZ}}^{\text{fn}}$. The t -integration in \mathcal{F}_1 in (5.31) disappears and we have $\mathcal{F}_1 = F_1$. In the similar way in the F-string case, we obtain the non-relativistic D-string action

$$\begin{aligned} S_{\text{NR}}^{D1} = & T_{\text{NR}} \int d^2\xi \sqrt{s \det g_0} \left[\frac{1}{2} g_0^{ij} \partial_i y^{\underline{A}} \partial_j y_{\underline{A}} + \frac{\lambda^2}{2} (my^2 - ny'^2) \right. \\ & \left. + 2i\bar{\theta}_- \gamma^i D_i \theta_- + \frac{1}{4} (F_1)_{ij} (F_1)^{ij} \right]. \end{aligned} \quad (5.37)$$

This is a free field action of scalars, fermions and a gauge field propagating on (2,0)- or (0,2)-brane worldvolume.

5.3 D3-brane

Thirdly, we consider a D3-brane for which $\varrho = \sigma_1$ and $\sigma = \sigma_3$. The gluing matrix is

$$M = \sqrt{-s} \Gamma^{\bar{A}_0 \dots \bar{A}_3} \otimes i\sigma_2. \quad (5.38)$$

Since

$$M' \Gamma_{\bar{B}_1 \dots \bar{B}_3} i\sigma_2 = \Gamma_{\bar{B}_1 \dots \bar{B}_3} i\sigma_2 M, \quad M' \Gamma_{\bar{A}} \varrho = -\Gamma_{\bar{A}} \varrho M, \quad M' \Gamma_{\bar{A}} \sigma = -\Gamma_{\bar{A}} \sigma M, \quad (5.39)$$

\mathcal{F} and h_3 are of order Ω as in (5.30) and the WZ part is expanded as

$$T h_5 = T_{\text{NR}} \Omega^{-2} h_5^{\text{div}} + T_{\text{NR}} h_5^{\text{fn}} + O(\Omega^4), \quad (5.40)$$

$$h_5^{\text{div}} = \frac{\sqrt{s}}{3!} \mathbf{L}_0^{\bar{A}_1} \mathbf{L}_0^{\bar{A}_2} \mathbf{L}_0^{\bar{A}_3} \bar{L}_{+0} \Gamma_{\bar{A}_1 \dots \bar{A}_3} i\sigma_2 L_{+0}, \quad (5.41)$$

$$h_5^{\text{fn}} = h_2^{(5)} + h_1^{(3)} \mathcal{F}_1 \quad (5.42)$$

with

$$\begin{aligned} h_2^{(5)} = & \frac{\sqrt{s}}{3!} \left[\mathbf{L}_0^{\bar{A}_1} \mathbf{L}_0^{\bar{A}_2} \mathbf{L}_0^{\bar{A}_3} \bar{L}_{-1} \Gamma_{\bar{A}_1 \dots \bar{A}_3} \sigma_1 L_{-1} + 3 \mathbf{L}_0^{\bar{A}_1} \mathbf{L}_0^{\bar{A}_2} \mathbf{L}_2^{\bar{A}_3} \bar{L}_{+0} \Gamma_{\bar{A}_1 \dots \bar{A}_3} \sigma_1 L_{+0} \right. \\ & + 2 \mathbf{L}_0^{\bar{A}_1} \mathbf{L}_0^{\bar{A}_2} \mathbf{L}_0^{\bar{A}_3} \bar{L}_{+0} \Gamma_{\bar{A}_1 \dots \bar{A}_3} \sigma_1 L_{+2} + 6 \mathbf{L}_0^{\bar{A}_1} \mathbf{L}_0^{\bar{A}_2} \mathbf{L}_1^{\bar{A}_3} \bar{L}_{+0} \Gamma_{\bar{A}_1 \dots \bar{A}_2 \bar{A}_3} \sigma_1 L_{-1} \\ & + 6 \mathbf{L}_0^{\bar{A}_1} \mathbf{L}_1^{\bar{A}_2} \mathbf{L}_1^{\bar{A}_3} \bar{L}_{+0} \Gamma_{\bar{A}_1 \bar{A}_2 \bar{A}_3} \sigma_1 L_{+0} \\ & \left. + 4i\lambda (\delta^{(3,1)} \epsilon_{\bar{a}_1 \bar{a}_2 \bar{a}_3 \underline{a}_4 \underline{a}_5} \mathbf{L}_0^{\bar{a}_1} \mathbf{L}_0^{\bar{a}_2} \mathbf{L}_0^{\bar{a}_3} \mathbf{L}_1^{\underline{a}_4} \mathbf{L}_1^{\underline{a}_5} \right. \\ & \left. - \delta^{(1,3)} \epsilon_{\bar{a}'_1 \bar{a}'_2 \bar{a}'_3 \underline{a}'_4 \underline{a}'_5} \mathbf{L}_0^{\bar{a}'_1} \mathbf{L}_0^{\bar{a}'_2} \mathbf{L}_0^{\bar{a}'_3} \mathbf{L}_1^{\underline{a}'_4} \mathbf{L}_1^{\underline{a}'_5} \right), \end{aligned} \quad (5.43)$$

$$h_1^{(3)} = \sqrt{s} \left[2 \mathbf{L}_0^{\bar{A}} \bar{L}_{+0} \Gamma_{\bar{A}} \varrho L_{-1} + \mathbf{L}_1^{\underline{A}} \bar{L}_{+0} \Gamma_{\underline{A}} \varrho L_{+0} \right], \quad (5.44)$$

where $\delta^{(m,n)} = 1$ for an (m,n) -brane and $\delta^{(m,n)} = 0$ for others. This implies that the bosonic 4-form $C^{(4)}$ is expanded as

$$TdC^{(4)} = T_{\text{NR}}\Omega^{-2}dC_0^{(4)} + T_{\text{NR}}dC_2^{(4)} + O(\Omega^4), \quad (5.45)$$

$$dC_0^{(4)} = 0, \quad (5.46)$$

$$dC_2^{(4)} = \frac{\sqrt{s}}{3!}4i\lambda(\delta^{(3,1)}\epsilon_{\bar{a}_1\bar{a}_2\bar{a}_3\bar{a}_4\bar{a}_5}e_0^{\bar{a}_1}e_0^{\bar{a}_2}e_0^{\bar{a}_3}e_1^{\bar{a}_4}e_1^{\bar{a}_5} - \delta^{(1,3)}\epsilon_{\bar{a}'_1\bar{a}'_2\bar{a}'_3\bar{a}'_4\bar{a}'_5}e_0^{\bar{a}'_1}e_0^{\bar{a}'_2}e_0^{\bar{a}'_3}e_1^{\bar{a}'_4}e_1^{\bar{a}'_5}). \quad (5.47)$$

On the other hand, as \mathcal{F} is of order Ω , the DBI part is expanded as in (5.33). As was done in the $p = 1$ case, we find

$$d(\sqrt{s \det g_0} d^4\xi) = d(\det((\mathbf{L}_0^{\bar{A}})_i)d^4\xi) = -\frac{\sqrt{s}}{3!}\mathbf{L}_0^{\bar{A}_1} \dots \mathbf{L}_0^{\bar{A}_3} \bar{L}_{+0} \Gamma_{\bar{A}_1 \dots \bar{A}_3} i\sigma_2 L_{+0}. \quad (5.48)$$

Thus the fermionic part contained in $\mathcal{L}_{\text{WZ}}^{\text{div}}$ and $\mathcal{L}_{\text{DBI}}^{\text{div}}$ cancel each other. In addition, the bosonic terms are deleted by choosing

$$C_0^{(4)} = -\frac{1}{4!}\epsilon_{\bar{A}_0 \dots \bar{A}_3} e_0^{\bar{A}_0} \dots e_0^{\bar{A}_3} \quad (5.49)$$

which satisfies $dC_0^{(4)} = 0$. Thus the matrix M leads to the consistent non-relativistic limit of the AdS D3-brane.

The non-relativistic D3-brane action is given as (5.9) with (5.35) and

$$\mathcal{L}_{\text{WZ}}^{\text{D3}} = \int_0^1 dt \left[C_2^{(4)} + C_1^{(2)} \hat{\mathcal{F}}_1 \right] + C_2^{(4)} \quad (5.50)$$

with

$$\begin{aligned} C_2^{(4)} &= 2c \left[\frac{1}{3!} \hat{\mathbf{L}}_0^{\bar{A}_1} \hat{\mathbf{L}}_0^{\bar{A}_2} \hat{\mathbf{L}}_0^{\bar{A}_3} (\hat{L}_{-1} \Gamma_{\bar{A}_1 \bar{A}_2 \bar{A}_3} i\sigma_2 \theta_- + \hat{L}_{+2} \Gamma_{\bar{A}_1 \bar{A}_2 \bar{A}_3} i\sigma_2 \theta_+) \right. \\ &\quad + \frac{1}{2} \hat{\mathbf{L}}_0^{\bar{A}_1} \hat{\mathbf{L}}_0^{\bar{A}_2} \hat{\mathbf{L}}_1^{\bar{A}_3} (\hat{L}_{-1} \Gamma_{\bar{A}_1 \bar{A}_2 \bar{A}_3} i\sigma_2 \theta_+ + \hat{L}_{+0} \Gamma_{\bar{A}_1 \bar{A}_2 \bar{A}_3} i\sigma_2 \theta_-) \\ &\quad \left. + \hat{\mathbf{L}}_0^{\bar{A}_1} \hat{\mathbf{L}}_0^{\bar{A}_2} \hat{\mathbf{L}}_2^{\bar{A}_3} \hat{L}_{+0} \Gamma_{\bar{A}_1 \bar{A}_2 \bar{A}_3} i\sigma_2 \theta_+ + \hat{\mathbf{L}}_0^{\bar{A}_1} \hat{\mathbf{L}}_1^{\bar{A}_2} \hat{\mathbf{L}}_1^{\bar{A}_3} \hat{L}_{+0} \Gamma_{\bar{A}_1 \bar{A}_2 \bar{A}_3} i\sigma_2 \theta_+ \right], \\ C_1^{(2)} &= 2c (\hat{\mathbf{L}}_0^{\bar{A}} \hat{L}_{+0} \Gamma_{\bar{A}} \varrho \theta_- + \hat{L}_{-1} \Gamma_{\bar{A}} \varrho \theta_+) + \hat{\mathbf{L}}_1^{\bar{A}} \hat{L}_{+0} \Gamma_{\bar{A}} \varrho \theta_+. \end{aligned} \quad (5.51)$$

The bosonic contribution $C_2^{(4)}$ is

$$\begin{aligned} \int_{\Sigma} C_2^{(4)} &= 4i\sqrt{s}\lambda \int_{\Sigma} \left[\delta^{(3,1)} \text{vol}_{\Sigma_3} \epsilon_{\underline{ab}} dy^{\underline{a}} y^{\underline{b}} - \delta^{(1,3)} \text{vol}_{\Sigma'_3} \epsilon_{\underline{a}'\underline{b}'} dy^{\underline{a}'} y^{\underline{b}'} \right], \\ &= 4i\sqrt{s}\lambda \int d^4\xi \sqrt{s \det g_0} \left[\delta^{(3,1)} \epsilon_{\underline{ab}} \partial_{\xi^i} y^{\underline{a}} y^{\underline{b}} - \delta^{(1,3)} \epsilon_{\underline{a}'\underline{b}'} \partial_{\xi^i} y^{\underline{a}'} y^{\underline{b}'} \right] \end{aligned} \quad (5.52)$$

where $\Sigma_m \times \Sigma'_n$ is the (m,n) -brane worldvolume, and $\text{vol}_{\Sigma_\ell} = \frac{1}{\ell!} \epsilon_{\bar{a}_1 \dots \bar{a}_\ell} e_0^{\bar{a}_1} \dots e_0^{\bar{a}_\ell}$. $\xi(\xi')$ represents the worldvolume direction in $\text{AdS}_5(S^5)$ respectively). By fixing the κ -symmetry as $\theta_+ = 0$, the non-relativistic action is simplified as

$$\begin{aligned} S_{\text{NR}}^{\text{D3}} &= T_{\text{NR}} \int d^4\xi \sqrt{s \det g_0} \left[\frac{1}{2} g_0^{ij} \partial_i y^{\underline{A}} \partial_j y^{\underline{B}} \eta_{\underline{A}\underline{B}} + \frac{\lambda^2}{2} (my^2 - ny'^2) \right. \\ &\quad \left. + 2i\bar{\theta} \gamma^i D_j \theta_- + \frac{1}{4} (F_1)_{ij} (F_1)^{ij} \right] + T_{\text{NR}} \int_{\Sigma} C_2^{(4)}. \end{aligned} \quad (5.53)$$

5.4 D5-brane

Fourthly, we consider a D5-brane for which $\varrho = \sigma_1$ and $\sigma = \sigma_3$. The gluing matrix is

$$M = \sqrt{-s}\Gamma^{\bar{A}_0 \dots \bar{A}_5} \otimes \rho, \quad \rho = \sigma_1, \sigma_3, 1. \quad (5.54)$$

Since

$$M'\Gamma_{\bar{B}_1 \dots \bar{B}_5} \varrho = \pm \Gamma_{\bar{B}_1 \dots \bar{B}_5} \varrho M, \quad M'\Gamma_{\bar{A}} \varrho = \pm \Gamma_{\bar{A}} \varrho M, \quad \rho = \begin{cases} \sigma_1, 1 \\ \sigma_3 \end{cases} \quad (5.55)$$

$$M'\Gamma_{\bar{B}_1 \dots \bar{B}_3} i\sigma_2 = \pm \Gamma_{\bar{B}_1 \dots \bar{B}_3} i\sigma_2 M, \quad \rho = \begin{cases} 1 \\ \sigma_1, \sigma_3 \end{cases} \quad (5.56)$$

$$M'\Gamma_{\bar{A}} \sigma = \pm \Gamma_{\bar{A}} \sigma M, \quad \rho = \begin{cases} \sigma_3, 1 \\ \sigma_1 \end{cases}, \quad (5.57)$$

\mathcal{F} is of order Ω only for $\rho = \sigma_1$. In this case the WZ part is expanded as

$$Th_7 = T_{\text{NR}} \Omega^{-2} h_7^{\text{div}} + T_{\text{NR}} h_7^{\text{fn}} + O(\Omega^4), \quad (5.58)$$

$$h_7^{\text{div}} = \frac{\sqrt{s}}{5!} \mathbf{L}_0^{\bar{A}_1} \dots \mathbf{L}_0^{\bar{A}_5} \bar{L}_{+0} \Gamma_{\bar{A}_1 \dots \bar{A}_5} \varrho L_{+0}, \quad (5.59)$$

$$h_7^{\text{fn}} = h_2^{(7)} + h_1^{(5)} \mathcal{F}_1 + \frac{1}{2} h_0^{(3)} \mathcal{F}_1^2 \quad (5.60)$$

and the DBI part is expanded as (5.33). We find that $h_2^{(7)}$, $h_1^{(5)}$ and $h_0^{(3)}$ are given as

$$\begin{aligned} h_2^{(7)} = & \frac{\sqrt{s}}{5!} \left[\mathbf{L}_0^{\bar{A}_1} \dots \mathbf{L}_0^{\bar{A}_5} \bar{L}_{-1} \Gamma_{\bar{A}_1 \dots \bar{A}_5} \varrho L_{-1} + 5 \mathbf{L}_0^{\bar{A}_1} \dots \mathbf{L}_0^{\bar{A}_4} \mathbf{L}_2^{\bar{A}_5} \bar{L}_{+0} \Gamma_{\bar{A}_1 \dots \bar{A}_5} \varrho L_{+0} \right. \\ & + 2 \mathbf{L}_0^{\bar{A}_1} \dots \mathbf{L}_0^{\bar{A}_5} \bar{L}_{+0} \Gamma_{\bar{A}_1 \dots \bar{A}_5} \varrho L_{+2} + 10 \mathbf{L}_0^{\bar{A}_1} \dots \mathbf{L}_0^{\bar{A}_4} \mathbf{L}_1^{\bar{A}_5} \bar{L}_{+0} \Gamma_{\bar{A}_1 \dots \bar{A}_4 \bar{A}_5} \varrho L_{-1} \\ & \left. + 20 \mathbf{L}_0^{\bar{A}_1} \dots \mathbf{L}_0^{\bar{A}_3} \mathbf{L}_1^{\bar{A}_4} \mathbf{L}_1^{\bar{A}_5} \bar{L}_{+0} \Gamma_{\bar{A}_1 \dots \bar{A}_3 \bar{A}_4 \bar{A}_5} \varrho L_{+0} \right], \quad (5.61) \end{aligned}$$

$$\begin{aligned} h_1^{(5)} = & \frac{\sqrt{s}}{3!} \left[2 \mathbf{L}_0^{\bar{A}_1} \mathbf{L}_0^{\bar{A}_2} \mathbf{L}_0^{\bar{A}_3} \bar{L}_{+0} \Gamma_{\bar{A}_1 \bar{A}_2 \bar{A}_3} i\sigma_2 L_{-1} + 3 \mathbf{L}_0^{\bar{A}_1} \mathbf{L}_0^{\bar{A}_2} \mathbf{L}_1^{\bar{A}_3} \bar{L}_{+0} \Gamma_{\bar{A}_1 \bar{A}_2 \bar{A}_3} i\sigma_2 L_{+0} \right. \\ & \left. + \delta^{(4,2)} i \lambda \epsilon_{\bar{a}_1 \dots \bar{a}_4 \bar{a}_5} \mathbf{L}_0^{\bar{a}_1} \dots \mathbf{L}_0^{\bar{a}_4} \mathbf{L}_1^{\bar{a}_5} - \delta^{(2,4)} i \lambda \epsilon_{\bar{a}'_1 \dots \bar{a}'_4 \bar{a}'_5} \mathbf{L}_0^{\bar{a}'_1} \dots \mathbf{L}_0^{\bar{a}'_4} \mathbf{L}_1^{\bar{a}'_5} \right], \quad (5.62) \end{aligned}$$

$$h_0^{(3)} = \sqrt{s} \mathbf{L}_0^{\bar{A}} \bar{L}_{+0} \Gamma_{\bar{A}} \varrho L_{+0}. \quad (5.63)$$

This implies that the bosonic 6-form $C^{(6)}$ is expanded as

$$TdC^{(6)} = T_{\text{NR}} \Omega^{-2} dC_0^{(6)} + T_{\text{NR}} dC_2^{(6)} + O(\Omega^4), \quad (5.64)$$

$$dC_0^{(6)} = 0, \quad (5.65)$$

$$dC_2^{(6)} = \frac{i\sqrt{s}}{3!} \lambda \left[\delta^{(4,2)} \epsilon_{\bar{a}_1 \dots \bar{a}_4 \bar{a}_5} e_0^{\bar{a}_1} \dots e_0^{\bar{a}_4} e_1^{\bar{a}_5} - \delta^{(2,4)} \epsilon_{\bar{a}'_1 \dots \bar{a}'_4 \bar{a}'_5} e_0^{\bar{a}'_1} \dots e_0^{\bar{a}'_4} e_1^{\bar{a}'_5} \right] F_1. \quad (5.66)$$

Because

$$d(\sqrt{s \det g_0} d^6 \xi) = -i \frac{\sqrt{s}}{5!} \mathbf{L}_0^{\bar{A}_1} \dots \mathbf{L}_0^{\bar{A}_5} \bar{L}_{+0} \Gamma_{\bar{A}_1 \dots \bar{A}_5} \varrho L_{+0}, \quad (5.67)$$

h_7^{div} in (5.59) and the fermionic term in the DBI part $\mathcal{L}_{\text{DBI}}^{div}$ cancel each other. As before, one sees that the bosonic terms are also deleted by choosing $C_0^{(6)} = -\frac{1}{6!}\epsilon_{\bar{A}_0\cdots\bar{A}_5}e_0^{\bar{A}_0}\cdots e_0^{\bar{A}_5}$. Summarizing we have shown that the matrix M with $\varrho = \sigma_1$ leads to the consistent non-relativistic limit of the AdS D5-brane.⁵

The non-relativistic D5-brane action is given as (5.9) with (5.35) and

$$\mathcal{L}_{\text{WZ}}^{\text{D5}} = \int_0^1 dt \left[C_2^{(6)} + C_1^{(4)} \hat{\mathcal{F}}_1 + \frac{1}{2} C_0^{(2)} \hat{\mathcal{F}}_1^2 \right] + C_2^{(6)} \quad (5.68)$$

with

$$\begin{aligned} C_2^{(6)} &= 2c \left[\frac{1}{5!} \hat{\mathbf{L}}_0^{\bar{A}_1} \cdots \hat{\mathbf{L}}_0^{\bar{A}_5} (\hat{L}_{-1} \Gamma_{\bar{A}_1 \cdots \bar{A}_5} \varrho \theta_- + \hat{L}_{+2} \Gamma_{\bar{A}_1 \cdots \bar{A}_5} \varrho \theta_+) \right. \\ &\quad + \frac{1}{4!} \hat{\mathbf{L}}_0^{\bar{A}_1} \cdots \hat{\mathbf{L}}_0^{\bar{A}_4} \hat{\mathbf{L}}_1^{\bar{A}_5} (\hat{L}_{-1} \Gamma_{\bar{A}_1 \cdots \bar{A}_4 \bar{A}_5} \varrho \theta_+ + \hat{L}_{+0} \Gamma_{\bar{A}_1 \cdots \bar{A}_4 \bar{A}_5} \varrho \theta_-) \\ &\quad + \frac{1}{4!} \hat{\mathbf{L}}_0^{\bar{A}_1} \cdots \hat{\mathbf{L}}_0^{\bar{A}_4} \hat{\mathbf{L}}_2^{\bar{A}_5} \hat{L}_{+0} \Gamma_{\bar{A}_1 \cdots \bar{A}_5} \varrho \theta_+ \\ &\quad \left. + \frac{1}{3!} \hat{\mathbf{L}}_0^{\bar{A}_1} \hat{\mathbf{L}}_0^{\bar{A}_2} \hat{\mathbf{L}}_0^{\bar{A}_3} \hat{\mathbf{L}}_1^{\bar{A}_4} \hat{\mathbf{L}}_1^{\bar{A}_5} \hat{L}_{+0} \Gamma_{\bar{A}_1 \bar{A}_2 \bar{A}_3 \bar{A}_4 \bar{A}_5} \varrho \theta_+ \right], \\ C_1^{(4)} &= 2c \left[\frac{1}{3!} \hat{\mathbf{L}}_0^{\bar{A}_1} \hat{\mathbf{L}}_0^{\bar{A}_2} \hat{\mathbf{L}}_0^{\bar{A}_3} (\hat{L}_{+0} \Gamma_{\bar{A}_1 \bar{A}_2 \bar{A}_3} i \sigma_2 \theta_- + \hat{L}_{-1} \Gamma_{\bar{A}_1 \bar{A}_2 \bar{A}_3} i \sigma_2 \theta_+) \right. \\ &\quad \left. + \frac{1}{2} \hat{\mathbf{L}}_0^{\bar{A}_1} \hat{\mathbf{L}}_0^{\bar{A}_2} \hat{\mathbf{L}}_1^{\bar{A}_3} \hat{L}_{+0} \Gamma_{\bar{A}_1 \bar{A}_2 \bar{A}_3} i \sigma_2 \theta_+ \right], \\ C_0^{(2)} &= 2c \left[\hat{\mathbf{L}}_0^{\bar{A}} \hat{L}_{+0} \Gamma_{\bar{A}} \varrho \theta_+ \right]. \end{aligned} \quad (5.69)$$

The bosonic contribution is

$$\begin{aligned} \int_{\Sigma} C_2^{(6)} &= 4i\sqrt{s}\lambda \int_{\Sigma} \left[\delta^{(4,2)} \text{vol}_{\Sigma_4} y F_1 - \delta^{(2,4)} \text{vol}_{\Sigma'_4} y' F_1 \right] \\ &= -4i\sqrt{s}\lambda \int d^6 \xi \sqrt{s \det g_0} \left[\delta^{(4,2)} \partial_i y (*A_1)^{i'} - \delta^{(2,4)} \partial_i y (*A_1)^i \right] \end{aligned} \quad (5.70)$$

where $y(y')$ is the transverse direction in $\text{AdS}_5(\text{S}^5)$, and $i(i')$ represents the worldvolume directions in $\text{AdS}_5(\text{S}^5)$. $*$ means the Hodge dual in Σ_2 or Σ'_2 . The κ -gauge symmetry is fixed by $\theta_+ = 0$, and the non-relativistic action is simplified as

$$\begin{aligned} S_{\text{NR}}^{\text{D5}} &= T_{\text{NR}} \int d^6 \xi \sqrt{s \det g_0} \left[\frac{1}{2} g_0^{ij} \partial_i y^{\underline{A}} \partial_j y_{\underline{A}} + \frac{\lambda^2}{2} (m y^2 - n y'^2) \right. \\ &\quad \left. + 2i\bar{\theta}_- \gamma^i D_i \theta_- + \frac{1}{4} (F_1)_{ij} (F_1)^{ij} \right] + T_{\text{NR}} \int_{\Sigma} C_2^{(6)}. \end{aligned} \quad (5.71)$$

5.5 D7-brane

Finally, let us consider a D7-brane for which $\varrho = \sigma_1$ and $\sigma = \sigma_3$. By using the gluing matrix

$$M = \sqrt{-s} \Gamma^{\bar{A}_0 \cdots \bar{A}_6} \otimes i \sigma_2 \quad (5.72)$$

⁵It is now obvious that for NS5-brane with $\varrho = \sigma_3$ and $\sigma = -\sigma_1$ the gluing matrix (5.54) with $\rho = \sigma_3$ leads to the consistent non-relativistic NS5-brane.

one derives

$$\begin{aligned}
 M'\Gamma_{\bar{B}_1\dots\bar{B}_7}i\sigma_2 &= \Gamma_{\bar{B}_1\dots\bar{B}_7}i\sigma_2 M, & M'\Gamma_{\bar{B}_1\dots\bar{B}_5}\varrho &= -\Gamma_{\bar{B}_1\dots\bar{B}_5}\varrho M, \\
 M'\Gamma_{\bar{B}_1\dots\bar{B}_3}i\sigma_2 &= \Gamma_{\bar{B}_1\dots\bar{B}_3}i\sigma_2 M, & M'\Gamma_{\bar{A}}\varrho &= -\Gamma_{\bar{A}}\varrho M, \\
 M'\Gamma_{\bar{A}}\sigma &= -\Gamma_{\bar{A}}\sigma M.
 \end{aligned} \tag{5.73}$$

These imply that \mathcal{F} is of order Ω and the WZ part is expanded as

$$Th_9 = T_{\text{NR}}\Omega^{-2}h_9^{\text{div}} + T_{\text{NR}}h_9^{\text{fin}} + O(\Omega^4), \tag{5.74}$$

$$h_9^{\text{div}} = \frac{\sqrt{s}}{7!}\mathbf{L}_0^{\bar{A}_1}\dots\mathbf{L}_0^{\bar{A}_7}\bar{L}_{+0}\Gamma_{\bar{A}_1\dots\bar{A}_7}i\sigma_2 L_{+0}, \tag{5.75}$$

$$h_9^{\text{fin}} = h_2^{(9)} + h_1^{(7)}\mathcal{F}_1 + \frac{1}{2}h_0^{(5)}\mathcal{F}_1^2, \tag{5.76}$$

and the DBI is as in (5.33). It is straightforward to see that $h_2^{(9)}$, $h_1^{(7)}$ and $h_0^{(5)}$ are given as

$$\begin{aligned}
 h_2^{(9)} &= \frac{\sqrt{s}}{7!}\left[\mathbf{L}_0^{\bar{A}_1}\dots\mathbf{L}_0^{\bar{A}_7}\bar{L}_{-1}\Gamma_{\bar{A}_1\dots\bar{A}_7}i\sigma_2 L_{-1} + 7\mathbf{L}_0^{\bar{A}_1}\dots\mathbf{L}_0^{\bar{A}_6}\mathbf{L}_2^{\bar{A}_7}\bar{L}_{+0}\Gamma_{\bar{A}_1\dots\bar{A}_7}i\sigma_2 L_{+0} \right. \\
 &\quad + 2\mathbf{L}_0^{\bar{A}_1}\dots\mathbf{L}_0^{\bar{A}_7}\bar{L}_{+0}\Gamma_{\bar{A}_1\dots\bar{A}_7}i\sigma_2 L_{+2} + 14\mathbf{L}_0^{\bar{A}_1}\dots\mathbf{L}_0^{\bar{A}_6}\mathbf{L}_1^{\bar{A}_7}\bar{L}_{+0}\Gamma_{\bar{A}_1\dots\bar{A}_6\bar{A}_7}i\sigma_2 L_{-1} \\
 &\quad \left. + 42\mathbf{L}_0^{\bar{A}_1}\dots\mathbf{L}_0^{\bar{A}_5}\mathbf{L}_1^{\bar{A}_6}\mathbf{L}_1^{\bar{A}_7}\bar{L}_{+0}\Gamma_{\bar{A}_1\dots\bar{A}_5\bar{A}_6\bar{A}_7}i\sigma_2 L_{+0}\right], \tag{5.77}
 \end{aligned}$$

$$h_1^{(7)} = \frac{\sqrt{s}}{5!}\left[2\mathbf{L}_0^{\bar{A}_1}\dots\mathbf{L}_0^{\bar{A}_5}\bar{L}_{+0}\Gamma_{\bar{A}_1\dots\bar{A}_5}\varrho L_{-1} + 5\mathbf{L}_0^{\bar{A}_1}\dots\mathbf{L}_0^{\bar{A}_4}\mathbf{L}_1^{\bar{A}_5}\bar{L}_{+0}\Gamma_{\bar{A}_1\dots\bar{A}_4\bar{A}_5}\varrho L_{+0}\right], \tag{5.78}$$

$$\begin{aligned}
 h_0^{(5)} &= \frac{\sqrt{s}}{3!}\left[\mathbf{L}_0^{\bar{A}_1}\dots\mathbf{L}_0^{\bar{A}_3}\bar{L}_{+0}\Gamma_{\bar{A}_1\dots\bar{A}_3}i\sigma_2 L_{+0} \right. \\
 &\quad \left. + \frac{i\lambda}{5}(\delta^{(5,3)}\epsilon_{\bar{a}_1\bar{a}_2\bar{a}_3\bar{a}_4\bar{a}_5}\mathbf{L}_0^{\bar{a}_1}\mathbf{L}_0^{\bar{a}_2}\mathbf{L}_0^{\bar{a}_3}\mathbf{L}_0^{\bar{a}_4}\mathbf{L}_0^{\bar{a}_5} - \delta^{(3,5)}\epsilon_{\bar{a}'_1\bar{a}'_2\bar{a}'_3\bar{a}'_4\bar{a}'_5}\mathbf{L}_0^{\bar{a}'_1}\mathbf{L}_0^{\bar{a}'_2}\mathbf{L}_0^{\bar{a}'_3}\mathbf{L}_0^{\bar{a}'_4}\mathbf{L}_0^{\bar{a}'_5})\right]. \tag{5.79}
 \end{aligned}$$

This implies that the bosonic 8-form $C^{(8)}$ is expanded as

$$TdC^{(8)} = T_{\text{NR}}\Omega^{-2}dC_0^{(8)} + T_{\text{NR}}dC_2^{(8)} + O(\Omega^4), \tag{5.80}$$

$$dC_0^{(8)} = 0, \tag{5.81}$$

$$dC_2^{(8)} = \frac{2i\sqrt{s}}{5!}\lambda\left[\delta^{(5,3)}\epsilon_{\bar{a}_1\dots\bar{a}_5}e_0^{\bar{a}_1}\dots e_0^{\bar{a}_5} - \delta^{(3,5)}\epsilon_{\bar{a}'_1\dots\bar{a}'_5}e_0^{\bar{a}'_1}\dots e_0^{\bar{a}'_5}\right](F_1)^2. \tag{5.82}$$

As before, we find

$$d(\sqrt{s \det g_0} d^8\xi) = d(\det((\mathbf{L}_0^{\bar{A}})_i)d^8\xi) = -i\frac{\sqrt{s}}{7!}\mathbf{L}_0^{\bar{A}_1}\dots\mathbf{L}_0^{\bar{A}_7}\bar{L}_{+0}\Gamma_{\bar{A}_1\dots\bar{A}_7}i\sigma_2 L_{+0}. \tag{5.83}$$

This implies that h_9^{div} and the fermionic terms in $\mathcal{L}_{\text{DBI}}^{\text{div}}$ cancel each other. The bosonic terms are also deleted by choosing $C_0^{(8)} = -\frac{1}{8!}\epsilon_{\bar{A}_0\dots\bar{A}_7}e_0^{\bar{A}_0}\dots e_0^{\bar{A}_7}$. Thus we find that the matrix M leads to the consistent non-relativistic limit of the AdS D7-brane.

The non-relativistic D7-brane action is given as (5.9) with (5.35) and

$$\mathcal{L}_{\text{WZ}}^{\text{D7}} = \int_0^1 dt \left[C_2^{(8)} + C_1^{(6)}\hat{\mathcal{F}}_1 + \frac{1}{2}C_0^{(4)}\hat{\mathcal{F}}_1^2 \right] + C_2^{(8)} \tag{5.84}$$

with

$$\begin{aligned}
C_2^{(8)} &= 2c \left[\frac{1}{7!} \hat{\mathbf{L}}_0^{\bar{A}_1} \cdots \hat{\mathbf{L}}_0^{\bar{A}_7} (\hat{L}_{-1} \Gamma_{\bar{A}_1 \dots \bar{A}_7} i\sigma_2 \theta_- + \hat{L}_{+2} \Gamma_{\bar{A}_1 \dots \bar{A}_7} i\sigma_2 \theta_+) \right. \\
&\quad + \frac{1}{6!} \hat{\mathbf{L}}_0^{\bar{A}_1} \cdots \hat{\mathbf{L}}_0^{\bar{A}_6} \hat{\mathbf{L}}_1^{\bar{A}_7} (\hat{L}_{-1} \Gamma_{\bar{A}_1 \dots \bar{A}_6 \bar{A}_7} i\sigma_2 \theta_+ + \hat{L}_{+0} \Gamma_{\bar{A}_1 \dots \bar{A}_6 \bar{A}_7} i\sigma_2 \theta_-) \\
&\quad + \frac{1}{6!} \hat{\mathbf{L}}_0^{\bar{A}_1} \cdots \hat{\mathbf{L}}_0^{\bar{A}_6} \hat{\mathbf{L}}_2^{\bar{A}_7} \hat{L}_{+0} \Gamma_{\bar{A}_1 \dots \bar{A}_7} i\sigma_2 \theta_+ \\
&\quad \left. + \frac{1}{5!} \hat{\mathbf{L}}_0^{\bar{A}_1} \cdots \hat{\mathbf{L}}_0^{\bar{A}_5} \hat{\mathbf{L}}_1^{\bar{A}_6} \hat{\mathbf{L}}_1^{\bar{A}_7} \hat{L}_{+0} \Gamma_{\bar{A}_1 \dots \bar{A}_5 \bar{A}_6 \bar{A}_7} i\sigma_2 \theta_+ \right], \\
C_1^{(6)} &= 2c \left[\frac{1}{5!} \hat{\mathbf{L}}_0^{\bar{A}_1} \cdots \hat{\mathbf{L}}_0^{\bar{A}_5} (\hat{L}_{+0} \Gamma_{\bar{A}_1 \dots \bar{A}_5} \varrho \theta_- + \hat{L}_{-1} \Gamma_{\bar{A}_1 \dots \bar{A}_5} \varrho \theta_+) \right. \\
&\quad \left. + \frac{1}{4!} \hat{\mathbf{L}}_0^{\bar{A}_1} \cdots \hat{\mathbf{L}}_0^{\bar{A}_4} \hat{\mathbf{L}}_1^{\bar{A}_5} \hat{L}_{+0} \Gamma_{\bar{A}_1 \dots \bar{A}_4 \bar{A}_5} \varrho \theta_+ \right], \\
C_0^{(4)} &= 2c \left[\frac{1}{3!} \hat{\mathbf{L}}_0^{\bar{A}_1} \cdots \hat{\mathbf{L}}_0^{\bar{A}_3} \hat{L}_{+0} \Gamma_{\bar{A}_1 \dots \bar{A}_3} i\sigma_2 \theta_+ \right]. \tag{5.85}
\end{aligned}$$

The bosonic contribution is

$$\begin{aligned}
\int_{\Sigma} C_2^{(8)} &= -2i\sqrt{s}\lambda \int_{\Sigma} \left[\delta^{(5,3)} \text{vol}_{\Sigma_5} A_1 F_1 - \delta^{(3,5)} \text{vol}_{\Sigma'_5} A_1 F_1 \right] \\
&= -2i\sqrt{s}\lambda \int d^8 \xi \sqrt{s \det g_0} \left[\delta^{(5,3)} (A_1)_{i'} (*F_1)^{i'} - \delta^{(3,5)} (A_1)_i (*F_1)^i \right] \tag{5.86}
\end{aligned}$$

where $i(i')$ represents worldvolume directions in $\text{AdS}_5(\mathbb{S}^5)$, and $*$ means the Hodge dual in Σ_3 or Σ'_3 . The κ -gauge symmetry is fixed by $\theta_+ = 0$, and then the non-relativistic action is simplified as

$$\begin{aligned}
S_{\text{NR}}^{D7} &= T_{\text{NR}} \int d^8 \xi \sqrt{s \det g_0} \left[\frac{1}{2} g_0^{ij} \partial_i y^A \partial_j y_A + \frac{\lambda^2}{2} (m y^2 - n y'^2) \right. \\
&\quad \left. + 2i\bar{\theta}_- \gamma^i D_i \theta_- + \frac{1}{4} (F_1)_{ij} (F_1)^{ij} \right] + T_{\text{NR}} \int_{\Sigma} C_2^{(8)}. \tag{5.87}
\end{aligned}$$

In summary, we have derived non-relativistic AdS Dp -brane actions in $\text{AdS}_5 \times \mathbb{S}^5$. In the flat limit $\lambda \rightarrow 0$, these actions for Lorentzian branes are reduced to the non-relativistic Dp -brane actions in flat spacetime derived in [12].

6. NH superalgebra of branes in $\text{AdS}_{4/7} \times \mathbb{S}^{7/4}$

The super-isometry algebra of the $\text{AdS}_{q+2} \times \mathbb{S}^{9-q}$ ($q = 2, 5$) solution of the eleven-dimensional supergravity is generated by translation P_A , Lorentz rotation $J_{AB} = (J_{ab}, J_{a'b'})$ and 32-component Majorana supercharge Q as

$$\begin{aligned}
[P_a, P_b] &= 4\epsilon^2 \lambda^2 J_{ab}, \quad [P_{a'}, P_{b'}] = -\epsilon^2 \lambda^2 J_{a'b'}, \\
[J_{ab}, P_c] &= \eta_{bc} P_a - \eta_{ac} P_b, \quad [J_{a'b'}, P_{c'}] = \eta_{b'c'} P_{a'} - \eta_{a'c'} P_{b'}, \\
[J_{ab}, J_{cd}] &= \eta_{bc} J_{ad} + \text{3-terms}, \quad [J_{a'b'}, J_{c'd'}] = \eta_{b'c'} J_{a'd'} + \text{3-terms}, \\
[P_A, Q] &= -\frac{\lambda}{2} Q \hat{\Gamma}_A, \quad [J_{AB}, Q] = \frac{1}{2} Q \Gamma_{AB}, \\
\{Q, Q\} &= -2C\Gamma^A P_A + \lambda C \hat{\Gamma}^{AB} J_{AB} \tag{6.1}
\end{aligned}$$

where $\widehat{\Gamma}^A = (2\mathcal{I}\Gamma^a, \mathcal{I}\Gamma^{a'})$ and $\widehat{\Gamma}^{AB} = (2\mathcal{I}\Gamma^{ab}, -\mathcal{I}\Gamma^{a'b'})$, and $\epsilon^2 = 1$ for $q = 2$ and $\epsilon^2 = -1$ for $q = 5$. For $q = 2$, this superalgebra is the super- $\text{AdS}_4 \times \text{S}^7$ algebra, $\text{osp}(8|4)$, with the vector index of AdS_4 , $a = 0, 1, 2, 3$ and that of S^7 , $a' = 4, 5, \dots, 9, \natural$. On the other hand, for $q = 5$, this superalgebra is the super- $\text{AdS}_7 \times \text{S}^4$ algebra, $\text{osp}(8^*|4)$ with the vector index of S^4 , $a = \natural, 1, 2, 3$ and that of AdS_7 , $a' = 4, 5, \dots, 9, 0$. We use the almost positive metric $\eta_{\mu\nu}$. We define λ and \mathcal{I} as

$$\lambda = \frac{1}{R}, \quad R^2 = 2kR_S^2 = \frac{1}{k}R_{\text{AdS}}^2, \quad k = \begin{cases} 1/2, & q = 2 \\ 2, & q = 5 \end{cases}$$

$$\mathcal{I} = \Gamma^{\natural 123}, \quad \Gamma^\natural = \begin{cases} \Gamma^0 & q = 2 \\ -\Gamma^\natural & q = 5 \end{cases} \quad (6.2)$$

where R_S and R_{AdS} are the radii of S^{9-q} and that of AdS_{q+2} , respectively. The gamma matrix $\Gamma^A \in \text{Spin}(1,10)$ and the charge conjugation matrix \mathcal{C} satisfy (2.2).

Letting g be a group element of the supergroup of the superalgebra (6.1), the LI Cartan one-form is defined as

$$\Omega = g^{-1}dg = \mathbf{L}^A P_A + \frac{1}{2}\mathbf{L}^{AB} J_{AB} + Q_\alpha L^\alpha. \quad (6.3)$$

The (anti-)commutation relations $[T_{\hat{A}}, T_{\hat{B}}] = f_{\hat{A}\hat{B}}^{\hat{C}} T_{\hat{C}}$, $T_{\hat{A}} = \{P_A, J_{AB}, Q_I\}$, are equivalent to the Maurer-Cartan (MC) equation

$$d\Omega = -\Omega \wedge \Omega. \quad (6.4)$$

The MC equations corresponding to the superalgebra (6.1) are derived as

$$\begin{aligned} d\mathbf{L}^A &= -\eta_{BC}\mathbf{L}^{AB}\mathbf{L}^C - \bar{L}\Gamma^A L, \\ d\mathbf{L}^{ab} &= -4\epsilon^2\lambda^2\mathbf{L}^a\mathbf{L}^b - \eta_{cd}\mathbf{L}^{ca}\mathbf{L}^{bd} + 2\lambda\bar{L}\mathcal{I}\Gamma^{ab}L, \\ d\mathbf{L}^{a'b'} &= +\epsilon^2\lambda^2\mathbf{L}^{a'}\mathbf{L}^{b'} - \eta_{c'd'}\mathbf{L}^{c'a'}\mathbf{L}^{b'd'} - \lambda\bar{L}\mathcal{I}\Gamma^{a'b'}L, \\ dL^\alpha &= \frac{\lambda}{2}\mathbf{L}^A\widehat{\Gamma}_A L - \frac{1}{4}\mathbf{L}^{AB}\Gamma_{AB}L. \end{aligned} \quad (6.5)$$

We introduce a matrix M

$$M = \ell\Gamma^{\bar{A}_0 \dots \bar{A}_p}, \quad \ell^2(-1)^{\lfloor \frac{p+1}{2} \rfloor} s = 1 \quad (6.6)$$

where $\{\bar{A}_0, \dots, \bar{A}_p\}$ are directions along which the brane worldvolume extends, so $A = (\bar{A}, \underline{A})$. Let an AdS brane extend along m directions in AdS_4 or S^4 and n directions in S^7 or AdS_7 , then the AdS brane worldvolume admits $\text{AdS}_m(\text{H}^m) \times \text{S}^n$ or $\text{S}^m \times \text{AdS}_n(\text{H}^n)$ isometry algebra for a Lorentzian (a Euclidean) brane, respectively. After contraction, the isometry of the transverse space is reduced to the Poincaré algebra $\text{iso}(4-m) \times \text{iso}(7-n)$ ($\text{iso}(3-m, 1) \times \text{iso}(7-n)$) or $\text{iso}(4-m) \times \text{iso}(7-n)$ ($\text{iso}(4-m) \times \text{iso}(6-n, 1)$) for a Lorentzian (a Euclidean) brane. We require that the contracted superalgebra contains a super subalgebra, the supersymmetrization of the direct product of the isometry algebra on the AdS brane worldvolume and the Lorentz symmetry in the transverse space, so($m -$

1-brane	2-brane	5-brane	6-brane	9-brane	10-brane
(1,1)	(0,3), (2,1)	(1,5), (3,3)	(0,7), (2,5), (4,3)	(3,7)	(4,7)

Table 4: Branes in $\text{AdS}_4 \times \text{S}^7$ and $\text{S}^4 \times \text{AdS}_7$

$1, 2) \times \text{so}(n+1) \times \text{so}(4-m) \times \text{so}(7-n)$ ($\text{so}(m, 1) \times \text{so}(n+1) \times \text{so}(3-m, 1) \times \text{so}(7-n)$) for a Lorentzian (a Euclidean) brane in $\text{AdS}_4 \times \text{S}^7$, and $\text{so}(m+1) \times \text{so}(n-1, 2) \times \text{so}(4-m) \times \text{so}(7-n)$ ($\text{so}(m+1) \times \text{so}(n, 1) \times \text{so}(4-m) \times \text{so}(6-n, 1)$) for a Lorentzian (a Euclidean) brane in $\text{S}^4 \times \text{AdS}_7$, respectively. This is satisfied if

$$M' \Gamma^{\bar{A}} = \Gamma^{\bar{A}} M, \quad M' \hat{\Gamma}^{\bar{A}\bar{B}} = \hat{\Gamma}^{\bar{A}\bar{B}} M, \quad (6.7)$$

where

$$M' = C^{-1} M^T C = (-1)^{p+1+\lfloor \frac{p+1}{2} \rfloor} M. \quad (6.8)$$

The first condition is satisfied if $p = 1, 2 \pmod{4}$. Since

$$M' \hat{\Gamma}^{\bar{A}\bar{B}} = (-1)^{p+1+d+\lfloor \frac{p+1}{2} \rfloor} \hat{\Gamma}^{\bar{A}\bar{B}} M \quad (6.9)$$

where d is the number of the Dirichlet directions contained in $\{\#, 1, 2, 3\}$, these are satisfied by (odd,odd)-branes ($p = 1 \pmod{4}$) and (even,odd)-branes ($p = 2 \pmod{4}$). We depict branes in table 4. The 10-brane is just $\text{AdS}_{4/7} \times \text{S}^{7/4}$ itself as $M = 1$. This table shows possible 1/2 supersymmetric subspaces which contain 1/2 BPS branes in $\text{AdS}_{4/7} \times \text{S}^{7/4}$. The p -branes with $p = 1 \pmod{4}$ are 1/2 BPS Dirichlet branes of an open supermembrane in $\text{AdS}_{4/7} \times \text{S}^{7/4}$ [26, 27]. The brane probe analysis for M-branes [42] is also consistent with this result.

We derive the NH superalgebra for these branes as IW contractions of the M pp-wave superalgebra. First, we rescale generators as

$$P_{\underline{A}} \rightarrow \frac{1}{\Omega} P_{\underline{A}}, \quad J_{\underline{A}\underline{B}} \rightarrow \frac{1}{\Omega} J_{\underline{A}\underline{B}}, \quad Q_{-} \rightarrow \frac{1}{\Omega} Q_{-} \quad (6.10)$$

where we have decomposed Q as

$$Q = Q_{+} + Q_{-}, \quad Q_{\pm} \mathcal{P}_{\pm} = Q_{\pm}, \quad \mathcal{P}_{\pm} = \frac{1}{2}(1 + M). \quad (6.11)$$

Substituting these into (6.1) and then taking the limit $\Omega \rightarrow 0$, we derive the NH superal-

gebra for an AdS brane

$$\begin{aligned}
[P_{\bar{a}}, P_{\bar{b}}] &= 4\epsilon^2 \lambda^2 J_{\bar{a}\bar{b}}, & [P_{\bar{a}'}, P_{\bar{b}'}] &= -\epsilon^2 \lambda^2 J_{\bar{a}'\bar{b}'}, \\
[P_{\bar{a}}, P_{\bar{b}}] &= 4\epsilon^2 \lambda^2 J_{\bar{a}\bar{b}}, & [P_{\bar{a}'}, P_{\bar{b}'}] &= -\epsilon^2 \lambda^2 J_{\bar{a}'\bar{b}'}, \\
[J_{\bar{A}\bar{B}}, P_{\bar{C}}] &= \eta_{\bar{B}\bar{C}} P_{\bar{A}} - \eta_{\bar{A}\bar{C}} P_{\bar{B}}, & [J_{\bar{A}\bar{B}}, P_{\bar{C}}] &= -\eta_{\bar{A}\bar{C}} P_{\bar{B}}, \\
[J_{\bar{A}\bar{B}}, P_{\bar{C}}] &= \eta_{\bar{B}\bar{C}} P_{\bar{A}} - \eta_{\bar{A}\bar{C}} P_{\bar{B}}, \\
[J_{\bar{A}\bar{B}}, J_{\bar{C}\bar{D}}] &= \eta_{\bar{B}\bar{C}} J_{\bar{A}\bar{D}} + 3\text{-terms}, & [J_{\bar{A}\bar{B}}, J_{\bar{C}\bar{D}}] &= \eta_{\bar{B}\bar{C}} J_{\bar{A}\bar{D}} + 3\text{-terms}, \\
[J_{\bar{A}\bar{B}}, J_{\bar{C}\bar{D}}] &= \eta_{\bar{B}\bar{C}} J_{\bar{A}\bar{D}} - \eta_{\bar{A}\bar{C}} J_{\bar{B}\bar{D}}, & [J_{\bar{A}\bar{B}}, J_{\bar{C}\bar{D}}] &= \eta_{\bar{B}\bar{D}} J_{\bar{C}\bar{A}} - \eta_{\bar{A}\bar{D}} J_{\bar{C}\bar{B}}, \\
[P_{\bar{A}}, Q_{\pm}] &= -\frac{\lambda}{2} Q_{\pm} \hat{\Gamma}_{\bar{A}}, & [P_{\bar{A}}, Q_{\pm}] &= -\frac{\lambda}{2} Q_{\pm} \hat{\Gamma}_{\bar{A}}, \\
[J_{\bar{A}\bar{B}}, Q_{\pm}] &= \frac{1}{2} Q_{\pm} \Gamma_{\bar{A}\bar{B}}, & [J_{\bar{A}\bar{B}}, Q_{\pm}] &= \frac{1}{2} Q_{\pm} \Gamma_{\bar{A}\bar{B}}, & [J_{\bar{A}\bar{B}}, Q_{\pm}] &= \frac{1}{2} Q_{\pm} \Gamma_{\bar{A}\bar{B}}, \\
\{Q_{+}, Q_{+}\} &= -2C\Gamma^{\bar{A}} \mathcal{P}_{+} P_{\bar{A}} + \lambda C \hat{\Gamma}^{\bar{A}\bar{B}} \mathcal{P}_{+} J_{\bar{A}\bar{B}} + \lambda C \hat{\Gamma}^{\bar{A}\bar{B}} \mathcal{P}_{+} J_{\bar{A}\bar{B}}, \\
\{Q_{+}, Q_{-}\} &= -2C\Gamma^{\bar{A}} \mathcal{P}_{-} P_{\bar{A}} + 2\lambda C \hat{\Gamma}^{\bar{A}\bar{B}} \mathcal{P}_{-} J_{\bar{A}\bar{B}}.
\end{aligned} \tag{6.12}$$

We note that this superalgebra contains two bosonic algebras and a superalgebra as subalgebras. One is the isometry of the (m, n) -brane worldvolume, generated by $\{P_{\bar{A}}, J_{\bar{A}\bar{B}}\}$, the $\text{AdS}_m(\mathbb{H}^m) \times S^n$ algebra for a Lorentzian (a Euclidean) brane in $\text{AdS}_4 \times S^7$ and the $S^m \times \text{AdS}_n(\mathbb{H}_n)$ algebra for a Lorentzian (a Euclidean) brane in $S^4 \times \text{AdS}_7$. Another is the isometry in the transverse space generated by $\{P_{\bar{A}}, J_{\bar{A}\bar{B}}\}$, the Poincaré algebra $\text{iso}(4-m) \times \text{iso}(7-n)$ ($\text{iso}(3-m, 1) \times \text{iso}(7-n)$) in $\text{AdS}_4 \times S^7$ and $\text{iso}(4-m) \times \text{iso}(7-n)$ ($\text{iso}(4-m) \times \text{iso}(6-n, 1)$) in $S^4 \times \text{AdS}_7$ for a Lorentzian (a Euclidean) brane. The other is the superalgebra generated by $\{P_{\bar{A}}, J_{\bar{A}\bar{B}}, J_{\bar{A}\bar{B}}, Q_{+}\}$

$$\begin{aligned}
[P_{\bar{a}}, P_{\bar{b}}] &= 4\epsilon^2 \lambda^2 J_{\bar{a}\bar{b}}, & [P_{\bar{a}'}, P_{\bar{b}'}] &= -\epsilon^2 \lambda^2 J_{\bar{a}'\bar{b}'}, & [J_{\bar{A}\bar{B}}, P_{\bar{C}}] &= \eta_{\bar{B}\bar{C}} P_{\bar{A}} - \eta_{\bar{A}\bar{C}} P_{\bar{B}}, \\
[J_{\bar{A}\bar{B}}, J_{\bar{C}\bar{D}}] &= \eta_{\bar{B}\bar{C}} J_{\bar{A}\bar{D}} + 3\text{-terms}, & [J_{\bar{A}\bar{B}}, J_{\bar{C}\bar{D}}] &= \eta_{\bar{B}\bar{C}} J_{\bar{A}\bar{D}} + 3\text{-terms}, \\
[P_{\bar{A}}, Q_{+}] &= -\frac{\lambda}{2} Q_{+} \hat{\Gamma}_{\bar{A}}, & [J_{\bar{A}\bar{B}}, Q_{+}] &= \frac{1}{2} Q_{+} \Gamma_{\bar{A}\bar{B}}, & [J_{\bar{A}\bar{B}}, Q_{+}] &= \frac{1}{2} Q_{+} \Gamma_{\bar{A}\bar{B}}, \\
\{Q_{+}, Q_{+}\} &= -2C\Gamma^{\bar{A}} \mathcal{P}_{+} P_{\bar{A}} + \lambda C \hat{\Gamma}^{\bar{A}\bar{B}} \mathcal{P}_{+} J_{\bar{A}\bar{B}} + \lambda C \hat{\Gamma}^{\bar{A}\bar{B}} \mathcal{P}_{+} J_{\bar{A}\bar{B}},
\end{aligned} \tag{6.13}$$

which is the supersymmetrization of the algebra, $\text{so}(m-1, 2) \times \text{so}(n+1) \times \text{so}(4-m) \times \text{so}(7-n)$ ($\text{so}(m, 1) \times \text{so}(n+1) \times \text{so}(3-m, 1) \times \text{so}(7-n)$) for a Lorentzian (a Euclidean) brane in $\text{AdS}_4 \times S^7$, and $\text{so}(m+1) \times \text{so}(n-1, 2) \times \text{so}(4-m) \times \text{so}(7-n)$ ($\text{so}(m+1) \times \text{so}(n, 1) \times \text{so}(4-m) \times \text{so}(6-n, 1)$) for a Lorentzian (a Euclidean) brane in $S^4 \times \text{AdS}_7$. For a $(4,7)$ -brane the superalgebra is obviously $\text{osp}(8|4)$ or $\text{osp}(8^*|4)$. Since the dimension of the bosonic subalgebra is 18 for $(0,3)$ - and $(3,3)$ -branes, 20 for $(1,1)$ -, $(2,1)$ -, $(1,5)$ - and $(2,5)$ -branes, 22 for a $(4,3)$ -brane, and 34 for $(0,7)$ - and $(3,7)$ -branes, one may guess the superalgebra as those including variants of $\text{osp}(4|2) \times \text{osp}(4|2)$, $\text{osp}(6|2) \times \text{so}(2|2)$, $\text{sp}(4|2) \times \text{osp}(4|2)$ and $\text{osp}(8|2) \times \text{su}(2)$, respectively. The existence of this superalgebra is ensured by (6.7).

The NH superalgebra (6.12) is equivalent to the MC equation

$$d\mathbf{L}^{\bar{A}} = -\eta_{\bar{B}\bar{C}}\mathbf{L}^{\bar{A}\bar{B}}\mathbf{L}^{\bar{C}} - \bar{L}_+\Gamma^{\bar{A}}L_+, \quad (6.14)$$

$$d\mathbf{L}^{\underline{A}} = -\eta_{\bar{B}\bar{C}}\mathbf{L}^{\bar{A}\bar{B}}\mathbf{L}^{\bar{C}} - \eta_{\underline{B}\underline{C}}\mathbf{L}^{\underline{A}\underline{B}}\mathbf{L}^{\underline{C}} - \bar{L}_+\Gamma^{\underline{A}}L_- - \bar{L}_-\Gamma^{\underline{A}}L_+, \quad (6.15)$$

$$d\mathbf{L}^{\bar{a}\bar{b}} = -4\epsilon^2\lambda^2\mathbf{L}^{\bar{a}}\mathbf{L}^{\bar{b}} - \eta_{\bar{c}\bar{d}}\mathbf{L}^{\bar{c}\bar{a}}\mathbf{L}^{\bar{b}\bar{d}} + 2\lambda\bar{L}_+\mathcal{I}\Gamma^{\bar{a}\bar{b}}L_+, \quad (6.16)$$

$$d\mathbf{L}^{\bar{a}'\bar{b}'} = +\lambda^2\mathbf{L}^{\bar{a}'}\mathbf{L}^{\bar{b}'} - \eta_{\bar{c}'\bar{d}'}\mathbf{L}^{\bar{c}'\bar{a}'}\mathbf{L}^{\bar{b}'\bar{d}'} - \lambda\bar{L}_+\mathcal{I}\Gamma^{\bar{a}'\bar{b}'}L_+, \quad (6.17)$$

$$d\mathbf{L}^{\underline{A}\underline{B}} = -\eta_{\underline{C}\underline{D}}\mathbf{L}^{\underline{C}\underline{D}}\mathbf{L}^{\underline{A}\underline{B}} + \lambda\bar{L}_+\hat{\Gamma}^{\underline{A}\underline{B}}L_+, \quad (6.18)$$

$$d\mathbf{L}^{\bar{a}\bar{b}} = -4\epsilon^2\lambda^2\mathbf{L}^{\bar{a}}\mathbf{L}^{\bar{b}} - \eta_{\bar{c}\bar{d}}\mathbf{L}^{\bar{c}\bar{a}}\mathbf{L}^{\bar{b}\bar{d}} - \eta_{\underline{c}\underline{d}}\mathbf{L}^{\bar{c}\bar{a}}\mathbf{L}^{\underline{b}\underline{d}} \\ + 2\lambda\bar{L}_+\mathcal{I}\Gamma^{\bar{a}\bar{b}}L_- + 2\lambda\bar{L}_-\mathcal{I}\Gamma^{\bar{a}\bar{b}}L_+, \quad (6.19)$$

$$d\mathbf{L}^{\bar{a}'\bar{b}'} = +\epsilon^2\lambda^2\mathbf{L}^{\bar{a}'}\mathbf{L}^{\bar{b}'} - \eta_{\bar{c}'\bar{d}'}\mathbf{L}^{\bar{c}'\bar{a}'}\mathbf{L}^{\bar{b}'\bar{d}'} - \eta_{\underline{c}'\underline{d}'}\mathbf{L}^{\bar{c}'\bar{a}'}\mathbf{L}^{\underline{b}'\underline{d}'} \\ - \lambda\bar{L}_+\mathcal{I}\Gamma^{\bar{a}'\bar{b}'}L_- - \lambda\bar{L}_-\mathcal{I}\Gamma^{\bar{a}'\bar{b}'}L_+, \quad (6.20)$$

$$dL_+ = \frac{\lambda}{2}\mathbf{L}^{\bar{A}}\hat{\Gamma}_{\bar{A}}L_+ - \frac{1}{4}\mathbf{L}^{\bar{A}\bar{B}}\Gamma_{\bar{A}\bar{B}}L_+ - \frac{1}{4}\mathbf{L}^{\underline{A}\underline{B}}\Gamma_{\underline{A}\underline{B}}L_+, \quad (6.21)$$

$$dL_- = \frac{\lambda}{2}\mathbf{L}^{\bar{A}}\hat{\Gamma}_{\bar{A}}L_- - \frac{\lambda}{2}\mathbf{L}^{\underline{A}}\hat{\Gamma}_{\underline{A}}L_+ \\ - \frac{1}{4}\mathbf{L}^{\bar{A}\bar{B}}\Gamma_{\bar{A}\bar{B}}L_- - \frac{1}{4}\mathbf{L}^{\underline{A}\underline{B}}\Gamma_{\underline{A}\underline{B}}L_- - \frac{1}{2}\mathbf{L}^{\bar{A}\bar{B}}\Gamma_{\bar{A}\bar{B}}L_+. \quad (6.22)$$

The MC equation above can be obtained by rescaling Cartan one-forms in (6.5) as

$$\mathbf{L}^{\underline{A}} \rightarrow \Omega\mathbf{L}^{\underline{A}}, \quad \mathbf{L}^{\bar{A}\bar{B}} \rightarrow \Omega\mathbf{L}^{\bar{A}\bar{B}}, \quad L_- \rightarrow \Omega L_-, \quad (6.23)$$

and taking the limit $\Omega \rightarrow 0$.

7. NH superalgebra of branes in M pp-wave

We define

$$P_{\pm} = \frac{1}{\sqrt{2}}(P_{\natural} \pm P_0), \quad P_i^* = \left(P_i^* = \begin{cases} J_{i0} \\ J_{i\natural} \end{cases}, P_{i'}^* = \begin{cases} J_{i'\natural} \\ J_{i'0} \end{cases} \right) \quad \text{for} \quad \begin{cases} \text{AdS}_4 \times \text{S}^7 \\ \text{AdS}_7 \times \text{S}^4 \end{cases} \\ Q^{(\pm)} = Q^{(\pm)}\ell_{\pm}, \quad \ell_{\pm} = \frac{1}{2}\Gamma_{\pm}\Gamma_{\mp}, \quad \Gamma_{\pm} = \frac{1}{\sqrt{2}}(\Gamma_{\natural} \pm \Gamma_0), \quad (7.1)$$

$$\mathcal{I} = \Gamma^{\natural 123}, \quad \Gamma^{\sharp} = \begin{cases} \Gamma^0 \\ -\Gamma^{\natural} \end{cases} \quad \text{for} \quad \begin{cases} \text{AdS}_4 \times \text{S}^7 \\ \text{AdS}_7 \times \text{S}^4 \end{cases} \quad (7.2)$$

where where $i = 1, 2, 3$ and $i' = 4, 5, 6, 7, 8, 9$.

Scaling generators in the super- $\text{AdS}_{4/7} \times \text{S}^{7/4}$ algebra as

$$P_+ \rightarrow \frac{1}{\Lambda^2}P_+, \quad P_i \rightarrow \frac{1}{\Lambda}P_i, \quad P_i^* \rightarrow \frac{1}{\Lambda}P_i^*, \quad Q^{(+)} \rightarrow \frac{1}{\Lambda}Q^{(+)} \quad (7.3)$$

and taking the limit $\Lambda \rightarrow 0$ limit [6], we obtain the M pp-wave superalgebra

$$\begin{aligned}
[P_-, P_i] &= \frac{4\lambda^2}{\sqrt{2}} P_i^*, & [P_-, P_{i'}] &= \frac{\lambda^2}{\sqrt{2}} P_{i'}^*, & [P_-, P_i^*] &= -\frac{1}{\sqrt{2}} P_i, \\
[P_i, P_j^*] &= \frac{1}{\sqrt{2}} \eta_{ij} P_+, & [P_i, J_{\hat{j}\hat{k}}] &= \eta_{ij} P_{\hat{k}} - \eta_{ik} P_j, & [P_i^*, J_{\hat{j}\hat{k}}] &= \eta_{ij} P_{\hat{k}}^* - \eta_{ik} P_j^*, \\
[J_{\hat{i}\hat{j}}, J_{\hat{k}\hat{l}}] &= \eta_{\hat{j}\hat{k}} J_{\hat{i}\hat{l}} + \text{3-terms}, \\
[P_-, Q^{(+)}] &= -\frac{3\lambda}{2\sqrt{2}} Q^{(+)} f, & [P_-, Q^{(-)}] &= -\frac{\lambda}{2\sqrt{2}} Q^{(-)} f, \\
[P_i, Q^{(-)}] &= -\frac{\lambda}{\sqrt{2}} Q^{(+)} f \Gamma_i \Gamma_+, & [P_{i'}, Q^{(-)}] &= -\frac{\lambda}{2\sqrt{2}} Q^{(+)} f \Gamma_{i'} \Gamma_+, \\
[J_{\hat{i}\hat{j}}, Q^{(\pm)}] &= \frac{1}{2} Q^{(\pm)} \Gamma_{\hat{i}\hat{j}}, & [P_i^*, Q^{(-)}] &= \frac{1}{2\sqrt{2}} Q^{(+)} \Gamma_i \Gamma_+, \\
\{Q^{(+)}, Q^{(+)}\} &= -2\mathcal{C} \Gamma_- P_+, \\
\{Q^{(-)}, Q^{(-)}\} &= -2\mathcal{C} \Gamma_+ P_- - \frac{\lambda}{\sqrt{2}} \mathcal{C} \hat{\Gamma}^{\hat{i}\hat{j}} J_{\hat{i}\hat{j}} \\
\{Q^{(\pm)}, Q^{(\mp)}\} &= -2\mathcal{C} \Gamma^{\hat{i}} \ell_{\mp} P_i - 4\lambda \mathcal{C} f \Gamma^i \ell_{\mp} P_i^* \mp 2\lambda \mathcal{C} f \Gamma^{i'} \ell_{\mp} P_{i'}^*,
\end{aligned} \tag{7.4}$$

where $\hat{\Gamma}^{\hat{i}\hat{j}} = (-2\Gamma_+ f \Gamma^{ij}, \Gamma_+ f \Gamma^{i'j'})$ and $f = \Gamma^{123}$. The bosonic subalgebra is the semi-direct product of the Heisenberg algebra generated by $\{P_i, P_i^*\}$ with an outer automorphism P_- and the Lorentz symmetry generated by $J_{\hat{i}\hat{j}}$.

7.1 Lorentzian branes

We consider a Lorentzian pp-wave brane for which $(+, -)$ directions are contained in the Neumann directions. We denote Neumann and Dirichlet directions, $\bar{A} = (+, -, \hat{i})$ and $\underline{A} = \hat{i}$, respectively.

We derive NH superalgebras of Lorentzian pp-wave branes as IW contractions of the M pp-wave superalgebra.

First we consider the bosonic subalgebra. The contraction is taken by rescaling generators as

$$P_{\underline{A}} \rightarrow \frac{1}{\Omega} P_{\underline{A}}, \quad J_{\bar{A}\bar{B}} \rightarrow \frac{1}{\Omega} J_{\bar{A}\bar{B}}, \quad P_{\hat{i}}^* \rightarrow \frac{1}{\Omega} P_{\hat{i}}^*, \tag{7.5}$$

and taking the limit $\Omega \rightarrow 0$. One obtains the NH algebra of an M pp-wave brane

$$\begin{aligned}
[P_-, P_i] &= \frac{4\lambda^2}{\sqrt{2}} P_i^*, & [P_-, P_{\hat{i}}] &= \frac{4\lambda^2}{\sqrt{2}} P_{\hat{i}}^*, & [P_-, P_{i'}] &= \frac{\lambda^2}{\sqrt{2}} P_{i'}^*, & [P_-, P_{i'}] &= \frac{\lambda^2}{\sqrt{2}} P_{i'}^*, \\
[P_-, P_i^*] &= -\frac{1}{\sqrt{2}} P_i, & [P_-, P_{\hat{i}}^*] &= -\frac{1}{\sqrt{2}} P_{\hat{i}}, & [P_i, P_j^*] &= \frac{1}{\sqrt{2}} \eta_{ij} P_+, \\
[P_i, J_{\hat{j}\hat{k}}] &= \eta_{ij} P_{\hat{k}}, & [P_i^*, J_{\hat{j}\hat{k}}] &= \eta_{ij} P_{\hat{k}}^*,
\end{aligned} \tag{7.6}$$

1-brane	2-brane	5-brane	6-brane	9-brane	10-brane
(+, -)	(+, -, 0, 1)	(+, -, 0, 4) (+, -, 2, 2)	(+, -, 0, 5) (+, -, 2, 3)	(+, -, 2, 6)	-

Table 5: Lorentzian M pp-wave branes

and

$$\begin{aligned}
 [P_{\hat{i}}, J_{\hat{j}\hat{k}}] &= \eta_{\hat{i}\hat{j}} P_{\hat{k}} - \eta_{\hat{i}\hat{k}} P_{\hat{j}}, & [P_{\hat{i}}, J_{\hat{j}\hat{k}}] &= \eta_{\hat{i}\hat{j}} P_{\hat{k}} - \eta_{\hat{i}\hat{k}} P_{\hat{j}}, \\
 [P_{\hat{i}}^*, J_{\hat{j}\hat{k}}] &= \eta_{\hat{i}\hat{j}} P_{\hat{k}}^* - \eta_{\hat{i}\hat{k}} P_{\hat{j}}^*, & [P_{\hat{i}}^*, J_{\hat{j}\hat{k}}] &= \eta_{\hat{i}\hat{j}} P_{\hat{k}}^* - \eta_{\hat{i}\hat{k}} P_{\hat{j}}^*, \\
 [J_{\hat{i}\hat{j}}, J_{\hat{k}\hat{l}}] &= \eta_{\hat{j}\hat{k}} J_{\hat{i}\hat{l}} + 3\text{-terms}, & [J_{\hat{i}\hat{j}}, J_{\hat{k}\hat{l}}] &= \eta_{\hat{j}\hat{k}} J_{\hat{i}\hat{l}} + 3\text{-terms}, \\
 [J_{\hat{i}\hat{j}}, J_{\hat{k}\hat{l}}] &= \eta_{\hat{j}\hat{k}} J_{\hat{i}\hat{l}} - \eta_{\hat{i}\hat{k}} J_{\hat{j}\hat{l}}, & [J_{\hat{i}\hat{j}}, J_{\hat{k}\hat{l}}] &= \eta_{\hat{i}\hat{l}} J_{\hat{j}\hat{k}} - \eta_{\hat{j}\hat{l}} J_{\hat{i}\hat{k}}.
 \end{aligned} \tag{7.7}$$

Next we consider the fermionic part. We decompose $Q^{(\bullet)}$ as

$$Q_{\pm}^{(\bullet)} = \pm Q_{\pm}^{(\bullet)} M \quad \text{with} \quad M = \ell \Gamma^{+-\bar{A}_1 \dots \bar{A}_{p-1}}, \quad M^2 = \ell^2 (-1)^{\lfloor \frac{p-1}{2} \rfloor} = 1 \tag{7.8}$$

which satisfies

$$M' = \mathcal{C}^{-1} M^T \mathcal{C} = (-1)^{p+1+\lfloor \frac{p+1}{2} \rfloor} M. \tag{7.9}$$

We demand that

$$M' \Gamma^{\bar{A}} = \Gamma^{\bar{A}} M, \tag{7.10}$$

$$M' \hat{\Gamma}^{\hat{i}\hat{j}} = \hat{\Gamma}^{\hat{i}\hat{j}} M. \tag{7.11}$$

Since

$$M' \Gamma^{\bar{A}} = (-1)^{1+\lfloor \frac{p+1}{2} \rfloor} \Gamma^{\bar{A}} M, \tag{7.12}$$

the first condition (7.10) is satisfied when $p = 1, 2 \pmod{4}$. The second condition (7.11) restricts the directions along which a pp-wave brane extends. Since

$$M' \hat{\Gamma}^{\hat{i}\hat{j}} = (-1)^{1+\lfloor \frac{p+1}{2} \rfloor + n} \hat{\Gamma}^{\hat{i}\hat{j}} M \tag{7.13}$$

where n is the number of the Neumann directions contained in $\{1, 2, 3\}$, we find that $n = \text{even}$ for $p = 1, 2 \pmod{4}$. In table 5 we summarize the result. This shows possible 1/2 supersymmetric subspaces in M pp-wave. Among them, the p -branes with $p = 1 \pmod{4}$ are Dirichlet branes of an open supermembrane in M pp-wave [46, 47].

Scaling $Q_{\pm}^{(\bullet)}$ as

$$Q_{-}^{(\bullet)} \rightarrow \frac{1}{\Omega} Q_{-}^{(\bullet)} \tag{7.14}$$

and taking the limit $\Omega \rightarrow 0$, we obtain the fermionic part of the NH superalgebra

$$\begin{aligned}
 [P_-, Q_\pm^{(+)}] &= -\frac{3\lambda}{2\sqrt{2}}Q_\pm^{(+)}f, & [P_-, Q_\pm^{(-)}] &= -\frac{\lambda}{2\sqrt{2}}Q_\pm^{(-)}f, \\
 [P_i, Q_\pm^{(-)}] &= -\frac{\lambda}{\sqrt{2}}Q_\pm^{(+)}f\Gamma_{\bar{i}}\Gamma_+, & [P_i, Q_+^{(-)}] &= -\frac{\lambda}{\sqrt{2}}Q_-^{(+)}f\Gamma_{\underline{i}}\Gamma_+, \\
 [P_{i'}, Q_\pm^{(-)}] &= -\frac{\lambda}{2\sqrt{2}}Q_\pm^{(+)}f\Gamma_{\bar{i}'}\Gamma_+, & [P_{i'}, Q_+^{(-)}] &= -\frac{\lambda}{2\sqrt{2}}Q_-^{(+)}f\Gamma_{\underline{i}'}\Gamma_+, \\
 [J_{\bar{i}\bar{j}}, Q_\pm^{(\bullet)}] &= \frac{1}{2}Q_\pm^{(\bullet)}\Gamma_{\bar{i}\bar{j}}, & [J_{\underline{i}\underline{j}}, Q_\pm^{(\bullet)}] &= \frac{1}{2}Q_\pm^{(\bullet)}\Gamma_{\underline{i}\underline{j}}, & [J_{\bar{i}\bar{j}}, Q_+^{(\pm)}] &= \frac{1}{2}Q_-^{(\pm)}\Gamma_{\bar{i}\bar{j}}, \\
 [P_i^*, Q_\pm^{(-)}] &= \frac{1}{2\sqrt{2}}Q_\pm^{(+)}\Gamma_{\bar{i}}\Gamma_+, & [P_i^*, Q_+^{(-)}] &= \frac{1}{2\sqrt{2}}Q_-^{(+)}\Gamma_{\underline{i}}\Gamma_+, \\
 \{Q_+^{(+)}, Q_+^{(+)}\} &= -2C\Gamma_-P_+P_+, \\
 \{Q_+^{(-)}, Q_+^{(-)}\} &= -2C\Gamma_+P_+P_- + \frac{\lambda}{\sqrt{2}}C\hat{\Gamma}^{\bar{i}\bar{j}}P_+J_{\bar{i}\bar{j}} + \frac{\lambda}{\sqrt{2}}C\hat{\Gamma}^{\underline{i}\underline{j}}P_+J_{\underline{i}\underline{j}}, \\
 \{Q_\pm^{(-)}, Q_\mp^{(-)}\} &= \sqrt{2}\lambda C\hat{\Gamma}^{\bar{i}\bar{j}}J_{\bar{i}\bar{j}}, \\
 \{Q_+^{(\pm)}, Q_+^{(\mp)}\} &= -2C\Gamma^{\bar{i}}\ell_\mp P_+P_{\bar{i}} - 4\lambda C f\Gamma^{\bar{i}}\ell_\mp P_+P_{\bar{i}}^* \mp 2\lambda C f\Gamma^{\bar{i}'}\ell_\mp P_+P_{i'}^*, \\
 \{Q_+^{(\pm)}, Q_-^{(\mp)}\} &= -2C\Gamma^{\underline{i}}\ell_\mp P_-P_{\underline{i}} - 4\lambda C f\Gamma^{\underline{i}}\ell_\mp P_-P_{\underline{i}}^* \mp 2\lambda C f\Gamma^{\underline{i}'}\ell_\mp P_-P_{i'}^*. \tag{7.15}
 \end{aligned}$$

Summarizing we have derived the NH superalgebra of an M pp-wave brane as (7.6), (7.7) and (7.15).

We note that the NH superalgebra of a Lorentzian M pp-wave brane contains a super-subalgebra generated by P_\pm , P_i , P_i^* , $J_{\bar{i}\bar{j}}$, $J_{\underline{i}\underline{j}}$ and $Q_+^{(\pm)}$

$$\begin{aligned}
 [P_-, P_i] &= \frac{4\lambda^2}{\sqrt{2}}P_i^*, & [P_-, P_{i'}] &= \frac{\lambda^2}{\sqrt{2}}P_{i'}^*, & [P_-, P_i^*] &= -\frac{1}{\sqrt{2}}P_i, \\
 [P_i, P_j^*] &= \frac{1}{\sqrt{2}}\eta_{ij}P_+, & [P_i, J_{\bar{j}\bar{k}}] &= \eta_{\bar{i}\bar{j}}P_{\bar{k}} - \eta_{\bar{i}\bar{k}}P_{\bar{j}}, & [P_i^*, J_{\bar{j}\bar{k}}] &= \eta_{\bar{i}\bar{j}}P_{\bar{k}}^* - \eta_{\bar{i}\bar{k}}P_{\bar{j}}^*, \\
 [J_{\bar{i}\bar{j}}, J_{\bar{k}\bar{l}}] &= \eta_{\bar{j}\bar{k}}J_{\bar{i}\bar{l}} + \text{3-terms}, & [J_{\underline{i}\underline{j}}, J_{\underline{k}\underline{l}}] &= \eta_{\underline{j}\underline{k}}J_{\underline{i}\underline{l}} + \text{3-terms}, \\
 [P_-, Q_+^{(+)}] &= -\frac{3\lambda}{2\sqrt{2}}Q_+^{(+)}f, & [P_-, Q_+^{(-)}] &= -\frac{\lambda}{2\sqrt{2}}Q_+^{(-)}f, \\
 [P_i, Q_+^{(-)}] &= -\frac{\lambda}{\sqrt{2}}Q_+^{(+)}f\Gamma_{\bar{i}}\Gamma_+, & [P_{i'}, Q_+^{(-)}] &= -\frac{\lambda}{2\sqrt{2}}Q_+^{(+)}f\Gamma_{\bar{i}'}\Gamma_+, \\
 [J_{\bar{i}\bar{j}}, Q_+^{(\bullet)}] &= \frac{1}{2}Q_+^{(\bullet)}\Gamma_{\bar{i}\bar{j}}, & [J_{\underline{i}\underline{j}}, Q_+^{(\bullet)}] &= \frac{1}{2}Q_+^{(\bullet)}\Gamma_{\underline{i}\underline{j}}, & [P_i^*, Q_+^{(-)}] &= \frac{1}{2\sqrt{2}}Q_+^{(+)}\Gamma_{\bar{i}}\Gamma_+, \\
 \{Q_+^{(+)}, Q_+^{(+)}\} &= -2C\Gamma_-P_+P_+, \\
 \{Q_+^{(-)}, Q_+^{(-)}\} &= -2C\Gamma_+P_+P_- + \frac{\lambda}{\sqrt{2}}C\hat{\Gamma}^{\bar{i}\bar{j}}P_+J_{\bar{i}\bar{j}} + \frac{\lambda}{\sqrt{2}}C\hat{\Gamma}^{\underline{i}\underline{j}}P_+J_{\underline{i}\underline{j}}, \\
 \{Q_+^{(\pm)}, Q_+^{(\mp)}\} &= -2C\Gamma^{\bar{i}}\ell_\mp P_+P_{\bar{i}} - 4\lambda C f\Gamma^{\bar{i}}\ell_\mp P_+P_{\bar{i}}^* \mp 2\lambda C f\Gamma^{\bar{i}'}\ell_\mp P_+P_{i'}^*. \tag{7.16}
 \end{aligned}$$

This is the supersymmetrization of the pp-wave algebra which is the isometry on the brane worldvolume and the Lorentz symmetry in the transverse space. The conditions (7.10) and (7.11) ensure the existence of this superalgebra.

1-brane	2-brane	5-brane	6-brane	9-brane	10-brane
(1,1)	(1,2), (3,0)	(1,5), (3,3)	(1,6), (3,4)	-	-

Table 6: Euclidean M pp-wave branes

7.2 Euclidean branes

We consider a Euclidean pp-wave brane for which $(+, -)$ directions are contained in the Dirichlet directions. We denote Neumann and Dirichlet directions as $\bar{A} = \hat{\bar{i}}$ and $\underline{A} = (+, -, \hat{\underline{i}})$, respectively.

We derive NH superalgebras of Euclidean pp-wave branes as IW contractions of the M pp-wave superalgebra. First we consider the bosonic subalgebra. The contraction is taken by rescaling generators as

$$P_{\underline{A}} \rightarrow \frac{1}{\Omega} P_{\underline{A}}, \quad J_{\bar{A}\bar{B}} \rightarrow \frac{1}{\Omega} J_{\bar{A}\bar{B}}, \quad P_{\hat{i}}^* \rightarrow \frac{1}{\Omega} P_{\hat{i}}^*, \quad (7.17)$$

and taking the limit $\Omega \rightarrow 0$. One obtains the NH algebra of a Euclidean M pp-wave brane

$$\begin{aligned} [P_-, P_{\hat{i}}] &= \frac{4\lambda^2}{\sqrt{2}} P_{\hat{i}}^*, & [P_-, P_{\hat{i}'}] &= \frac{\lambda^2}{\sqrt{2}} P_{\hat{i}'}^*, & [P_-, P_{\hat{i}}^*] &= -\frac{1}{\sqrt{2}} P_{\hat{i}}, & [P_{\hat{i}}, J_{\hat{j}\hat{k}}] &= \eta_{\hat{i}\hat{j}} P_{\hat{k}}, \\ [P_{\hat{i}}, P_{\hat{j}}^*] &= \frac{1}{\sqrt{2}} \eta_{\hat{i}\hat{j}} P_+, & [P_{\hat{i}}, P_{\hat{j}}^*] &= \frac{1}{\sqrt{2}} \eta_{\hat{i}\hat{j}} P_+, & [P_{\hat{i}}^*, J_{\hat{j}\hat{k}}] &= -\eta_{\hat{i}\hat{k}} P_{\hat{j}}^*, \end{aligned} \quad (7.18)$$

and (7.7).

Next we consider the fermionic part of the NH superalgebra. We decompose $Q^{(\bullet)}$ as

$$Q_{\pm}^{(\bullet)} = \pm Q_{\pm}^{(\bullet)} M, \quad M = \ell \Gamma^{\bar{A}_0 \dots \bar{A}_p}, \quad M^2 = \ell^2 (-1)^{\lfloor \frac{p+1}{2} \rfloor} = 1. \quad (7.19)$$

We demand that the conditions (7.10) and (7.11) are satisfied. The first condition (7.10) implies that $p = 1, 2 \pmod{4}$ as

$$M' \Gamma^{\bar{A}} = (-1)^{1 + \lfloor \frac{p+1}{2} \rfloor} \Gamma^{\bar{A}} M. \quad (7.20)$$

On the other hand, since

$$M' \hat{\Gamma}^{\hat{i}\hat{j}} = (-1)^{n + \lfloor \frac{p+1}{2} \rfloor} \hat{\Gamma}^{\hat{i}\hat{j}} M \quad (7.21)$$

where n is the number of the Neumann directions contained in $\{1, 2, 3\}$, the second condition is satisfied when $n = \text{odd}$ for $p = 1, 2 \pmod{4}$. We summarize the result in table 6. Among these $1/2$ supersymmetric subspaces, the p -branes with $p = 1 \pmod{4}$ are Dirichlet branes of an open supermembrane in M pp-wave [46, 47].

Scaling $Q_{\pm}^{(\bullet)}$ as (3.17) and taking the limit $\Omega \rightarrow 0$, we obtain the fermionic part of the NH superalgebra of a Euclidean M pp-wave brane

$$\begin{aligned}
 [P_-, Q_+^{(+)}] &= -\frac{3\lambda}{2\sqrt{2}}Q_-^{(+)}f, & [P_-, Q_+^{(-)}] &= -\frac{\lambda}{2\sqrt{2}}Q_-^{(-)}f, \\
 [P_{\hat{i}}, Q_{\pm}^{(-)}] &= -\frac{\lambda}{\sqrt{2}}Q_{\pm}^{(+)}f\Gamma_{\hat{i}}\Gamma_+, & [P_{\hat{i}}, Q_+^{(-)}] &= -\frac{\lambda}{\sqrt{2}}Q_-^{(+)}f\Gamma_{\hat{i}}\Gamma_+, \\
 [P_{\hat{i}'}, Q_{\pm}^{(-)}] &= -\frac{\lambda}{2\sqrt{2}}Q_{\pm}^{(+)}f\Gamma_{\hat{i}'}\Gamma_+, & [P_{\hat{i}'}, Q_+^{(-)}] &= -\frac{\lambda}{2\sqrt{2}}Q_-^{(+)}f\Gamma_{\hat{i}'}\Gamma_+, \\
 [J_{\hat{i}\hat{j}}, Q_{\pm}^{\bullet}] &= \frac{1}{2}Q_{\pm}^{(\bullet)}\Gamma_{\hat{i}\hat{j}}, & [J_{\hat{i}\hat{j}}, Q_{\pm}^{\bullet}] &= \frac{1}{2}Q_{\pm}^{(\bullet)}\Gamma_{\hat{i}\hat{j}}, & [J_{\hat{i}\hat{j}}, Q_+^{\bullet}] &= \frac{1}{2}Q_-^{(\bullet)}\Gamma_{\hat{i}\hat{j}}, \\
 [P_{\hat{i}}^*, Q_+^{(-)}] &= \frac{1}{2\sqrt{2}}Q_-^{(+)}\Gamma_{\hat{i}}\Gamma_+, & [P_{\hat{i}}^*, Q_{\pm}^{(-)}] &= \frac{1}{2\sqrt{2}}Q_{\pm}^{(+)}\Gamma_{\hat{i}}\Gamma_+, \\
 \{Q_{\pm}^{(+)}, Q_{\mp}^{(+)}\} &= -2\mathcal{C}\Gamma_- \mathcal{P}_{\mp} P_+, \\
 \{Q_+^{(-)}, Q_+^{(-)}\} &= -\frac{\lambda}{\sqrt{2}}\mathcal{C}\hat{\Gamma}^{\hat{i}\hat{j}}J_{\hat{i}\hat{j}} - \frac{\lambda}{\sqrt{2}}\mathcal{C}\hat{\Gamma}^{\hat{i}\hat{j}}J_{\hat{i}\hat{j}}, \\
 \{Q_{\pm}^{(-)}, Q_{\mp}^{(-)}\} &= -2\mathcal{C}\Gamma_+ \mathcal{P}_{\mp} P_- - \sqrt{2}\lambda\mathcal{C}\hat{\Gamma}^{\hat{i}\hat{j}}J_{\hat{i}\hat{j}}, \\
 \{Q_+^{(\pm)}, Q_+^{(\mp)}\} &= -2\mathcal{C}\Gamma_{\mp}^{\hat{i}}\ell_{\mp} \mathcal{P}_+ P_{\hat{i}} - 4\lambda\mathcal{C}f\Gamma_{\mp}^{\hat{i}}\ell_{\mp} \mathcal{P}_+ P_{\hat{i}}^* \mp 2\lambda\mathcal{C}f\Gamma_{\mp}^{\hat{i}'}\ell_{\mp} \mathcal{P}_+ P_{\hat{i}'}^*, \\
 \{Q_+^{(\pm)}, Q_-^{(\mp)}\} &= -2\mathcal{C}\Gamma_{\mp}^{\hat{i}}\ell_{\mp} \mathcal{P}_- P_{\hat{i}} - 4\lambda\mathcal{C}f\Gamma_{\mp}^{\hat{i}}\ell_{\mp} \mathcal{P}_- P_{\hat{i}}^* \mp 2\lambda\mathcal{C}f\Gamma_{\mp}^{\hat{i}'}\ell_{\mp} \mathcal{P}_- P_{\hat{i}'}^*. \tag{7.22}
 \end{aligned}$$

Summarizing we have derived the NH superalgebra of a Euclidean M pp-wave brane as (7.18), (7.7) and (7.22).

We note that there exists a super-subalgebra of the NH superalgebra generated by $P_{\hat{i}}$, $P_{\hat{i}}^*$, $J_{\hat{i}\hat{j}}$, $J_{\hat{i}\hat{j}}$ and $Q_+^{(\pm)}$

$$\begin{aligned}
 [P_{\hat{i}}, J_{\hat{j}\hat{k}}] &= \eta_{\hat{i}\hat{j}}P_{\hat{k}} - \eta_{\hat{i}\hat{k}}P_{\hat{j}}, & [P_{\hat{i}}^*, J_{\hat{j}\hat{k}}] &= \eta_{\hat{i}\hat{j}}P_{\hat{k}}^* - \eta_{\hat{i}\hat{k}}P_{\hat{j}}^*, \\
 [J_{\hat{i}\hat{j}}, J_{\hat{k}\hat{l}}] &= \eta_{\hat{j}\hat{k}}J_{\hat{i}\hat{l}} + \text{3-terms}, & [J_{\hat{i}\hat{j}}, J_{\hat{k}\hat{l}}] &= \eta_{\hat{j}\hat{k}}J_{\hat{i}\hat{l}} + \text{3-terms}, \\
 [P_{\hat{i}}, Q_+^{(-)}] &= -\frac{\lambda}{\sqrt{2}}Q_+^{(+)}f\Gamma_{\hat{i}}\Gamma_+, & [P_{\hat{i}'}, Q_+^{(-)}] &= -\frac{\lambda}{2\sqrt{2}}Q_+^{(+)}f\Gamma_{\hat{i}'}\Gamma_+, \\
 [J_{\hat{i}\hat{j}}, Q_+^{\bullet}] &= \frac{1}{2}Q_+^{(\bullet)}\Gamma_{\hat{i}\hat{j}}, & [J_{\hat{i}\hat{j}}, Q_+^{\bullet}] &= \frac{1}{2}Q_+^{(\bullet)}\Gamma_{\hat{i}\hat{j}}, & [P_{\hat{i}}^*, Q_+^{(-)}] &= \frac{1}{2\sqrt{2}}Q_+^{(+)}\Gamma_{\hat{i}}\Gamma_+, \\
 \{Q_+^{(-)}, Q_+^{(-)}\} &= -\frac{\lambda}{\sqrt{2}}\mathcal{C}\hat{\Gamma}^{\hat{i}\hat{j}}J_{\hat{i}\hat{j}} - \frac{\lambda}{\sqrt{2}}\mathcal{C}\hat{\Gamma}^{\hat{i}\hat{j}}J_{\hat{i}\hat{j}}, \\
 \{Q_+^{(\pm)}, Q_+^{(\mp)}\} &= -2\mathcal{C}\Gamma_{\mp}^{\hat{i}}\ell_{\mp} \mathcal{P}_+ P_{\hat{i}} - 4\lambda\mathcal{C}f\Gamma_{\mp}^{\hat{i}}\ell_{\mp} \mathcal{P}_+ P_{\hat{i}}^* \mp 2\lambda\mathcal{C}f\Gamma_{\mp}^{\hat{i}'}\ell_{\mp} \mathcal{P}_+ P_{\hat{i}'}^*. \tag{7.23}
 \end{aligned}$$

This is the supersymmetrization of the Poincaré algebra generated by $\{P_{\hat{i}}, J_{\hat{i}\hat{j}}\}$ which is the isometry on the brane worldvolume and the Lorentz symmetry in the transverse space generated by $\{P_{\hat{i}}^*, J_{\hat{i}\hat{j}}\}$. The conditions (7.10) and (7.11) ensure the existence of this super-subalgebra.

8. Branes in $\text{AdS}_{4/7} \times \text{S}^{7/4}$

The action for an M2-brane [48] is composed of the NG action and the WZ action

$$S = T \int_{\Sigma} [\mathcal{L}_{\text{NG}} + \mathcal{L}_{\text{WZ}}], \quad \mathcal{L}_{\text{NG}} = d^{p+1} \xi \sqrt{s \det g_{ij}}, \quad (8.1)$$

where $s = -1$ for a Lorentzian brane and $s = 1$ for a Euclidean brane. g_{ij} is given by $g_{ij} = \mathbf{L}_i^A \mathbf{L}_j^B \eta_{AB}$ with $\mathbf{L}_i^A = \partial_i Z^{\hat{M}} \mathbf{L}_{\hat{M}}^A$. T is the tension of the brane. For an M5-brane case, the self-duality of the two-form gauge field B on the brane is imposed on the field equations, or the NG action is replaced by the PST action [49]

$$\begin{aligned} \mathcal{L}_{\text{PST}} &= \sqrt{s \det(g_{ij} - i\alpha^2 \mathcal{H}_{ij}^*)} + \alpha^2 \frac{\sqrt{s \det g}}{4} \mathcal{H}^{*ij} \mathcal{H}_{ij}, \\ \mathcal{H}_{ij} &= \mathcal{H}_{ijk} v^k, \quad \mathcal{H}^{*ij} = \mathcal{H}^{*ijk} v_k, \quad v_i = \frac{\partial_i a}{\sqrt{g^{jk} \partial_j a \partial_k a}}, \\ \mathcal{H} &= H + \mathcal{C}_3, \quad \mathcal{H}^{*ijk} = \frac{1}{3! \sqrt{s \det g}} \epsilon^{ijklmn} \mathcal{H}_{lmn}, \quad H = dB \end{aligned} \quad (8.2)$$

where \mathcal{C}_3 is a pullback of the three-form gauge field, and $\alpha^2 = i\sqrt{s}$. Here the PST scalar field a is contained in the M5-brane case as a modification of the usual DBI action. The WZ term is known to be characterized by manifestly supersymmetric $(p+2)$ -form $h_{p+2} = d\mathcal{L}_{\text{WZ}}$, which is composed of the pullback of the supercurrents, \mathbf{L}^A and L^α , on the supergroup manifold and the modified field strength \mathcal{H} . The $(p+2)$ -form h_{p+2} is closed but not exact on the superspace, because \mathcal{L}_{WZ} is not superinvariant but quasi-superinvariant. Expanding h_{p+2} with respect to \mathcal{H}

$$h_{p+2}(\mathbf{L}^A, L^\alpha, \mathcal{H}) = h^{(p+2)}(\mathbf{L}^A, L^\alpha) - \frac{c}{2} \mathcal{H} h^{(p-1)}(\mathbf{L}^A, L^\alpha), \quad (8.3)$$

where c is a constant determined below, the closedness condition $dh_{p+2} = 0$ is expressed as

$$dh^{(p-1)} = 0, \quad (8.4)$$

$$dh^{(p+2)} - \frac{c}{2} d\mathcal{H} h^{(p-1)} = 0. \quad (8.5)$$

8.1 CE-cohomology classification

We show that M p -brane actions in $\text{AdS}_{q+2} \times \text{S}^{9-q}$ ($q = 2, 5$) can be classified as non-trivial elements of the CE-cohomology on the differential algebra, MC equations (6.5) for the super- $\text{AdS}_{q+2} \times \text{S}^{9-q}$ algebra.

In order to avoid an additional dimensional parameter, we assign dimensions as

$$\begin{array}{cccc} \mathbf{L}^A & L^\alpha & \lambda & \mathcal{H} \\ \dim & 1 & 1/2 & -1 \end{array} \quad (8.6)$$

For structureless branes, the dimension of S_{WZ} must be equal to the dimension of S_{NG} , from which we find $\dim h_{p+2} = p + 1$ because $\dim h_{p+2} = \dim \mathcal{L}_{\text{WZ}} = \dim \mathcal{L}_{\text{NG}} = p + 1$, and thus $\dim h^{(k)} = k - 1$. $h^{(k)}$ is composed of \mathbf{L}^A , L^α and λ , and thus we can write

$h^{(k)}$ as $\lambda^l (\mathbf{L}^A)^m (L^\alpha)^n$. The integers l, m and n are restricted by the properties of $h^{(k)}$, $\dim h^{(k)} = k - 1$ and $\deg h^{(k)} = k$, as

$$-l + m + \frac{1}{2}n = k - 1, \quad m + n = k. \quad (8.7)$$

For consistent flat limit, we demand $l \geq 0$ because λ is related to the inverse of the radii of AdS_{q+2} and S^{9-q} . In addition, we require that $\epsilon_{a_1 \dots a_4}$ and $\epsilon_{a'_1 \dots a'_7}$ are accompanied with λ ; $\lambda \epsilon_{a_1 \dots a_4}$ and $\lambda \epsilon_{a'_1 \dots a'_7}$, because $\epsilon_{a_1 \dots a_4}$ and $\epsilon_{a'_1 \dots a'_7}$ disappear in the flat limit. Noting that (8.7) implies $l = 1 - \frac{1}{2}n \leq 1$, we consider the cases with $l = 0$ and 1. It is easy to see that $(m, n) = (k - 2, 2)$ for $l = 0$ while $(m, n) = (k, 0)$ for $l = 1$. In the former case, terms of the form $\mathbf{L}^{A_1} \dots \mathbf{L}^{A_{k-2}} \bar{L} \Gamma_{A_1 \dots A_{k-2}} L$ are candidates for $h^{(k)}$. These terms are non-trivial only for $k = 3, 4 \pmod{4}$, because $C\Gamma_{A_1 \dots A_{k-2}}$ is symmetric if $k - 2 = 1, 2 \pmod{4}$. In the latter case, $\lambda \epsilon_{a_1 \dots a_4} \mathbf{L}^{a_1} \dots \mathbf{L}^{a_4}$ and $\lambda \epsilon_{a'_1 \dots a'_7} \mathbf{L}^{a'_1} \dots \mathbf{L}^{a'_7}$ are candidates for $h^{(4)}$ and $h^{(7)}$, respectively. We summarize the non-trivial candidates for $h^{(k)}$

$$h^{(3)} : \mathbf{L}^A \bar{L} \Gamma_A L \quad (8.8)$$

$$h^{(4)} : \mathbf{L}^A \mathbf{L}^B \bar{L} \Gamma_{AB} L, \quad \lambda \epsilon_{a_1 \dots a_4} \mathbf{L}^{a_1} \dots \mathbf{L}^{a_4} \quad (8.9)$$

$$h^{(7)} : \mathbf{L}^{A_1} \dots \mathbf{L}^{A_5} \bar{L} \Gamma_{A_1 \dots A_5} L, \quad \lambda \epsilon_{a'_1 \dots a'_7} \mathbf{L}^{a'_1} \dots \mathbf{L}^{a'_7} \quad (8.10)$$

$$h^{(8)} : \mathbf{L}^{A_1} \dots \mathbf{L}^{A_6} \bar{L} \Gamma_{A_1 \dots A_6} L \quad (8.11)$$

where $\mathbf{L}^A \bar{L} \Gamma_A L$ stands for two candidates $\mathbf{L}^a \bar{L} \Gamma_a L$ and $\mathbf{L}^{a'} \bar{L} \Gamma_{a'} L$, and so on. For example, $h^{(4)}$ is of the form

$$h^{(4)} = c_1 \mathbf{L}^a \mathbf{L}^b \bar{L} \Gamma_{ab} L + c_2 \mathbf{L}^a \mathbf{L}^{a'} \bar{L} \Gamma_{aa'} L + c_3 \mathbf{L}^{a'} \mathbf{L}^{b'} \bar{L} \Gamma_{a'b'} L + c_4 \lambda \epsilon_{a_1 \dots a_4} \mathbf{L}^{a_1} \dots \mathbf{L}^{a_4}. \quad (8.12)$$

Next we are going to find $h^{(k)}$ satisfying (8.4) and (8.5). The first step for this is to find a closed form $dh^{(k)} = 0$ in (8.4). $h^{(k)}$ can be a closed form only when $k = 4$. This is due to the Fierz identity

$$(C\Gamma_{AB})_{(\alpha\beta} (C\Gamma^B)_{\gamma\delta)} = 0. \quad (8.13)$$

The coefficients are fixed by the closedness condition $dh^{(4)} = 0$ as

$$h^{(4)} = c \left[\frac{1}{2} \mathbf{L}^A \mathbf{L}^B \bar{L} \Gamma_{AB} L - \frac{6\lambda}{4!} \epsilon_{a_1 \dots a_4} \mathbf{L}^{a_1} \dots \mathbf{L}^{a_4} \right] \quad (8.14)$$

where $\epsilon_{0123} = -\epsilon_{\bar{1}\bar{2}\bar{3}} = +1$. As seen in appendix B, the overall coefficient c is fixed by the requirement of the κ -invariance [48, 50] of the total action S as $c = -1$ for Lorentzian brane and $c = i$ for Euclidean brane: $c = i\sqrt{s}$. Using $h^{(4)}$ above, the closed four-form h_4 is constructed as

$$h_4 = h^{(4)}. \quad (8.15)$$

Because h_4 is not exact on the superspace as will be shown below, we find that the M2-brane action in $AdS_{4/7} \times S^{7/4}$ is a non-trivial element of CE cohomology of the differential algebra (6.5), MC equations for super- $AdS_{4/7} \times S^{7/4}$ algebra. The obtained action is consistent with one given in [50].

Next we introduce $d\mathcal{H}$ to the differential algebra (6.5). Since $\mathcal{H} = dB + \mathcal{C}_3$ and $h_4 = c'd\mathcal{C}_3$ with a constant c' , $d\mathcal{H}$ is given by

$$c'd\mathcal{H} = h_4 = h^{(4)} \tag{8.16}$$

where $h^{(4)}$ is given in (8.14). If we can construct $h^{(7)}$ satisfying (8.5), then h_7 turns out to be a closed seven-form. We find that using (8.14) and (8.16) the condition (8.5) fixes coefficients of a linear combination of candidates as

$$h^{(7)} = c^2 \left[\frac{1}{5!} \mathbf{L}^{A_1} \dots \mathbf{L}^{A_5} \bar{L} \Gamma_{A_1 \dots A_5} L - \frac{6\lambda}{7!} \epsilon_{a'_1 \dots a'_7} \mathbf{L}^{a'_1} \dots \mathbf{L}^{a'_7} \right] \tag{8.17}$$

where $\epsilon_{4\dots 9\ddot{4}} = -\epsilon_{4\dots 90} = +1$ and $c' \equiv -c$. To see this we have used the Fierz identity,

$$(\mathcal{C}\Gamma^{A_1 \dots A_5})_{(\alpha\beta} (\mathcal{C}\Gamma_{A_5})_{\gamma\delta)} - 3(\mathcal{C}\Gamma^{[A_1 A_2})_{(\alpha\beta} (\mathcal{C}\Gamma^{A_3 A_4]})_{\gamma\delta)} = 0. \tag{8.18}$$

The closed seven-form is constructed using (8.14) and (8.17) as

$$h_7 = h^{(7)} - \frac{c}{2} h^{(4)} \mathcal{H}. \tag{8.19}$$

Because h_7 is not exact on the superspace as will be shown below, we find that M5-brane action in $\text{AdS}_{4/7} \times S^{7/4}$ is characterized as a non-trivial element of CE cohomology on the differential algebra (6.5) and (8.16). The constant c^2 is determined by the requirement that the total action is κ -invariant [49, 51] as $c^2 = -1$ and i for Lorentzian and Euclidean branes respectively, i.e. $c^2 = \alpha^2 = i\sqrt{s}$. See appendix B. The obtained action is consistent with one given in [51].

We show that the four- and seven-forms obtained above are not exact. Suppose that h_4 is exact, then there must exist $b^{(3)}$ such that $h^{(4)} = db^{(3)}$. $b^{(3)}$ can be written as $\lambda^l (\mathbf{L}^A)^m (L^\alpha)^n$ where integers l, m and n are restricted by the properties of $b^{(3)}$, $\dim b^{(3)} = 3$ and $\deg b^{(3)} = 3$. This implies that $l \leq 0$. We find that there is no candidate for $l = 0$. For $l = -1$, we find two candidates, $\lambda^{-1} \mathbf{L}^a \bar{L} \Gamma_a L$ and $\lambda^{-1} \mathbf{L}^{a'} \bar{L} \Gamma_{a'} L$, but any linear combination of them does not satisfy $h^{(4)} = db^{(3)}$. It is obvious that terms with $l \leq -2$ do not satisfy $h^{(4)} = db^{(3)}$. Thus, h_4 is not exact. Next, suppose that h_7 is exact, then there exists b_6 such that $h_7 = db_6$. This implies, expanding $b_6(\mathbf{L}^\mu, L^\alpha, \mathcal{H})$ as $b^{(6)}(\mathbf{L}^A, L^\alpha) + \frac{1}{2} \mathcal{H} b^{(3)}(\mathbf{L}^A, L^\alpha)$, that

$$h^{(7)} = db^{(6)} - \frac{c}{2} d\mathcal{H} b^{(3)}, \quad h^{(4)} = -db^{(3)}. \tag{8.20}$$

Because we have shown that $h^{(4)}$ is not exact, there dose not exist $b^{(3)}$ satisfying (8.20). Thus, we have shown that h_7 is not exact.

Summarizing we find that actions of M2- and M5-branes in $\text{AdS}_{4/7} \times S^{7/4}$ are characterized as non-trivial elements of the CE cohomology.

8.2 $(p + 1)$ -dimensional form of the WZ term

We derive $(p + 1)$ -dimensional form of the WZ-term following [51].

The supervielbein and super spin connection are given in appendix A.2. These satisfy the following differential equations

$$\partial_t \hat{\mathbf{L}}^A = -2\bar{\theta}\Gamma^A \hat{L}, \quad (8.21)$$

$$\partial_t \hat{\mathbf{L}}^{AB} = 2\lambda\bar{\theta}\hat{\Gamma}^{AB} \hat{L}, \quad (8.22)$$

$$\partial_t \hat{L} = d\theta - \frac{\lambda}{2} \hat{\mathbf{L}}^A \hat{\Gamma}_A \theta + \frac{1}{4} \hat{\mathbf{L}}^{AB} \Gamma_{AB} \theta, \quad (8.23)$$

where the symbols with “hat” imply that the fermionic variable θ is rescaled as $\theta \rightarrow t\theta$.

By using these equations, one finds that

$$\partial_t \hat{h}_4 = db_3, \quad b_3 = -c \hat{\mathbf{L}}^A \hat{\mathbf{L}}^B \hat{L} \Gamma_{AB} \theta. \quad (8.24)$$

This implies that

$$h_4 = d\mathcal{C}_3, \quad \mathcal{C}_3 = \int_0^1 dt b_3 + C^{(3)} \quad (8.25)$$

where $C^{(3)}$ is a bosonic 3-form satisfying $dC^{(3)} = h_4|_{\text{bosonic}}$. It follows from

$$-c\mathcal{H} = dB + \int_0^1 dt b_3 + C^{(3)} \quad (8.26)$$

that

$$-c\partial_t \hat{\mathcal{H}} = b_3. \quad (8.27)$$

In the similar way, one derives

$$\partial_t \hat{h}_7 = d(b_6 - \frac{c}{2} b_3 \hat{\mathcal{H}}), \quad b_6 = c^2 \frac{2}{5!} \hat{\mathbf{L}}^{A_1} \dots \hat{\mathbf{L}}^{A_5} \hat{L} \Gamma_{A_1 \dots A_5} \theta \quad (8.28)$$

so that

$$h_7 = d\mathcal{C}_6, \quad \mathcal{C}_6 = \int_0^1 dt (b_6 - \frac{c}{2} b_3 \hat{\mathcal{H}}) + C^{(6)} \quad (8.29)$$

where $C^{(6)}$ is a bosonic 6-form satisfying $dC^{(6)} = h_7|_{\text{bosonic}}$.

Summarizing the $(p+1)$ -dimensional form of the WZ term is given as

$$\mathcal{L}_{\text{WZ}}^{M2} = -i\sqrt{s} \int_0^1 dt \hat{\mathbf{L}}^A \hat{\mathbf{L}}^B \hat{L} \Gamma_{AB} \theta + C^{(3)}, \quad (8.30)$$

$$\mathcal{L}_{\text{WZ}}^{M5} = i\sqrt{s} \int_0^1 dt \left(\frac{2}{5!} \hat{\mathbf{L}}^{A_1} \dots \hat{\mathbf{L}}^{A_5} \hat{L} \Gamma_{A_1 \dots A_5} \theta - \frac{1}{2} \hat{\mathbf{L}}^A \hat{\mathbf{L}}^B \hat{L} \Gamma_{AB} \theta \hat{\mathcal{H}} \right) + C^{(6)}. \quad (8.31)$$

9. Non-relativistic branes in $\text{AdS}_{4/7} \times \text{S}^{7/4}$

We consider the non-relativistic limit of the branes in $\text{AdS}_{4/7} \times \text{S}^{7/4}$ obtained in the previous section.

We scale coordinates and the tension as

$$X^{\underline{A}} \rightarrow \Omega X^{\underline{A}}, \quad \theta_- \rightarrow \Omega \theta_-, \quad (9.1)$$

$$T = T_{\text{NR}} \Omega^{-2}, \quad H = \Omega H_1. \quad (9.2)$$

(9.1) is consistent with (6.3). As can be seen from the concrete expression of the supercurrents given in appendix A.2, the supercurrents are expanded as (5.3). Expanding \mathbf{L}^{AB} as in (5.4) and substituting (5.3) and (5.4) into the MC equation (6.5), one finds that Cartan one forms $\{\mathbf{L}_m^{\bar{A}}, \mathbf{L}_m^{\underline{A}}, \mathbf{L}_m^{\bar{A}\bar{B}}, \mathbf{L}_m^{\underline{A}\underline{B}}, \mathbf{L}_m^{\underline{A}\bar{B}}, L_{\pm m} \mid m = 0, 1\}$ form the MC equation (6.14)-(6.22).

9.1 M2-brane

First we consider an M2-brane. The NG part \mathcal{L}_{NG} is expanded as in (5.15) with (5.16) and (5.17). By using (6.14) and $L_+ = ML_+$ with (6.6), one derives

$$d\mathcal{L}_{\text{NG}}^{\text{div}} = d(\det(\mathbf{L}_0^{\bar{A}})_i d^3\xi) = -\frac{i\sqrt{s}}{2!} \mathbf{L}_0^{\bar{A}_1} \mathbf{L}_0^{\bar{A}_2} \bar{L}_{+0} \Gamma_{\bar{A}_1 \bar{A}_2} L_{+0}. \quad (9.3)$$

Since

$$M' \Gamma^{\bar{A}\bar{B}} = \Gamma^{\bar{A}\bar{B}} M, \quad (9.4)$$

the four-form h_4 is expanded as

$$Th_4 = T_{\text{NR}} \Omega^{-2} h_4^{\text{div}} + T_{\text{NR}} h_4^{\text{fin}} + O(\Omega^4), \quad (9.5)$$

with

$$h_4^{\text{div}} = \frac{i\sqrt{s}}{2} \mathbf{L}_0^{\bar{A}} \mathbf{L}_0^{\bar{B}} \bar{L}_{+0} \Gamma_{\bar{A}\bar{B}} L_{+0}, \quad (9.6)$$

$$\begin{aligned} h_4^{\text{fin}} = & \frac{i\sqrt{s}}{2} \left[\mathbf{L}_0^{\bar{A}} \mathbf{L}_0^{\bar{B}} \bar{L}_{-1} \Gamma_{\bar{A}\bar{B}} L_{-1} + 2\mathbf{L}_0^{\bar{A}} \mathbf{L}_0^{\bar{B}} \bar{L}_{+0} \Gamma_{\bar{A}\bar{B}} L_{+2} + 2\mathbf{L}_0^{\bar{A}} \mathbf{L}_2^{\bar{B}} \bar{L}_{+0} \Gamma_{\bar{A}\bar{B}} L_{+0} \right. \\ & + 4\mathbf{L}_0^{\bar{A}} \mathbf{L}_1^{\bar{B}} \bar{L}_{+0} \Gamma_{\bar{A}\bar{B}} L_{-1} + \mathbf{L}_1^{\underline{A}} \mathbf{L}_1^{\underline{B}} \bar{L}_{+0} \Gamma_{\underline{A}\underline{B}} L_{+0} \\ & \left. - 6\lambda \delta^{(2,1)} \epsilon_{\bar{a}_1 \bar{a}_2 \underline{a}_3 \underline{a}_4} e_0^{\bar{a}_1} e_0^{\bar{a}_2} e_1^{\underline{a}_3} e_1^{\underline{a}_4} \right]. \quad (9.7) \end{aligned}$$

This implies that the bosonic 3-form C^3 is expanded as

$$TdC^{(3)} = T_{\text{NR}} \Omega^{-2} dC_0^{(3)} + T_{\text{NR}} dC_2^{(3)} + O(\Omega^2), \quad (9.8)$$

$$dC_0^{(3)} = 0, \quad (9.9)$$

$$dC_2^{(3)} = -3i\sqrt{s}\lambda \delta^{(2,1)} \epsilon_{\bar{a}_1 \bar{a}_2 \underline{a}_3 \underline{a}_4} e_0^{\bar{a}_1} e_0^{\bar{a}_2} e_1^{\underline{a}_3} e_1^{\underline{a}_4}. \quad (9.10)$$

Since

$$d(\det(\mathbf{L}_0^{\bar{A}})_i d^3\xi) + h_4^{\text{div}} = 0, \quad (9.11)$$

the fermionic contribution of $\mathcal{L}_{\text{NG}}^{\text{div}}$ and $\mathcal{L}_{\text{WZ}}^{\text{div}}$ cancel each other. In order to delete the bosonic terms of $\mathcal{L}_{\text{NG}}^{\text{div}} + \mathcal{L}_{\text{WZ}}^{\text{div}}$, we choose

$$C_0^{(3)} = -\frac{1}{3!} \epsilon_{\bar{A}_0 \bar{A}_1 \bar{A}_2} e_0^{\bar{A}_0} e_0^{\bar{A}_1} e_0^{\bar{A}_2}. \quad (9.12)$$

It follows from the expressions given in appendix A.2 that $dC_0^{(3)} = 0$. In summary, we find that the gluing matrix M leads to the consistent non-relativistic limit of the M2-brane in $\text{AdS}_{4/7} \times \text{S}^{7/4}$.

The non-relativistic M2-brane action is given as

$$S_{\text{NR}} = T_{\text{NR}} \int_{\Sigma} [\mathcal{L}_{\text{NG}}^{\text{fn}} + \mathcal{L}_{\text{WZ}}^{\text{fn}}] \quad (9.13)$$

with (5.17) and

$$\begin{aligned} \mathcal{L}_{\text{WZ}}^{\text{fn}} = & -i\sqrt{s} \int_0^1 dt \left[\hat{\mathbf{L}}_0^{\bar{A}} \hat{\mathbf{L}}_0^{\bar{B}} (\hat{L}_{-1} \Gamma_{\bar{A}\bar{B}} \theta_- + \hat{L}_{+2} \Gamma_{\bar{A}\bar{B}} \theta_+) + 2\hat{\mathbf{L}}_0^{\bar{A}} \hat{\mathbf{L}}_2^{\bar{B}} \hat{L}_{+0} \Gamma_{\bar{A}\bar{B}} \theta_+ \right. \\ & \left. + 2\hat{\mathbf{L}}_0^{\bar{A}} \hat{\mathbf{L}}_1^{\bar{B}} (\hat{L}_{-1} \Gamma_{\bar{A}\bar{B}} \theta_+ + \hat{L}_{+0} \Gamma_{\bar{A}\bar{B}} \theta_-) + \hat{\mathbf{L}}_1^{\bar{A}} \hat{\mathbf{L}}_1^{\bar{B}} \hat{L}_{+0} \Gamma_{\bar{A}\bar{B}} \theta_+ \right] + C_2^{(3)} \end{aligned} \quad (9.14)$$

The bosonic contribution is

$$\begin{aligned} \int_{\Sigma} C_2^{(3)} &= -3i\sqrt{s}\lambda\delta^{(2,1)} \int_{\Sigma} \text{vol}_{\Sigma_2} \epsilon_{\underline{ab}} y^{\underline{a}} dy^{\underline{b}} \\ &= -3i\sqrt{s}\lambda\delta^{(2,1)} \int d^3\xi \sqrt{s \det g_0} \epsilon_{\underline{ab}} y^{\underline{a}} \partial_{i'} y^{\underline{b}} \end{aligned}$$

where i' represent worldvolume directions in S^7 or AdS_7 .

The t -integration is easily done after fixing the κ -gauge symmetry by $\theta_+ = 0$ (see appendix B), which leads to

$$\begin{aligned} \mathbf{L}_0^{\bar{A}} &= e_0^{\bar{A}}, \quad \mathbf{L}_2^{\bar{A}} = e_2^{\bar{A}} - \bar{\theta}_- \Gamma^{\bar{A}} \text{D}\theta_-, \quad \mathbf{L}_1^{\bar{A}} = e_1^{\bar{A}}, \\ L_{-1} &= \text{D}\theta_- = d\theta_- - \frac{\lambda}{2} e_0^{\bar{A}} \hat{\Gamma}_{\bar{A}} \theta_- + \frac{1}{4} \omega^{\bar{A}\bar{B}} \Gamma_{\bar{A}\bar{B}} \theta_-, \\ (g_0)_{ij} &= (e_0^{\bar{A}})_i (e_0^{\bar{B}})_j \eta_{\bar{A}\bar{B}}. \end{aligned} \quad (9.15)$$

$(e_0^{\bar{A}})_i$ is the vielbein on the worldvolume in the static gauge, $x^{\bar{A}} = \xi^i$. Substituting these into the non-relativistic action we obtain

$$\begin{aligned} S_{\text{NR}} &= T_{\text{NR}} \int d^3\xi \sqrt{s \det g_0} \left[g_0^{ij} \partial_i y^{\bar{A}} \partial_j y^{\bar{B}} \eta_{\bar{A}\bar{B}} + \frac{\epsilon^2 \lambda^2}{2} (4my^2 - ny'^2) - 2\bar{\theta}_- \gamma^i \text{D}_i \theta_- \right] \\ &+ T_{\text{NR}} \int_{\Sigma} C_2^{(3)}. \end{aligned} \quad (9.16)$$

In the flat limit $\lambda \rightarrow 0$, this reproduces the non-relativistic action given in [11].

9.2 M5-brane

Next we consider an M5-brane for which $c^2 = i\sqrt{s}$. In this case the gluing matrix

$$M = \sqrt{-s} \Gamma^{\bar{A}_0 \dots \bar{A}_5}, \quad M' = -M \quad (9.17)$$

satisfies

$$M' \Gamma_{\bar{B}_1 \bar{B}_2} = -\Gamma_{\bar{B}_1 \bar{B}_2} M, \quad (9.18)$$

so that \mathcal{H} is of order Ω

$$\begin{aligned} \mathcal{H} &= \Omega \mathcal{H}_1 + O(\Omega^3), \\ \mathcal{H}_1 &= H_1 + \int_0^1 dt \left[\hat{\mathbf{L}}_0^{\bar{A}} \hat{\mathbf{L}}_0^{\bar{B}} (\hat{L}_{+0} \Gamma_{\bar{A}\bar{B}} \theta_- + \hat{L}_{-1} \Gamma_{\bar{A}\bar{B}} \theta_+) + 2 \hat{\mathbf{L}}_0^{\bar{A}} \hat{\mathbf{L}}_1^{\bar{B}} \hat{L}_{+0} \Gamma_{\bar{A}\bar{B}} \theta_+ \right]. \end{aligned} \quad (9.19)$$

The PST part \mathcal{L}_{PST} is expanded as

$$T\mathcal{L}_{\text{PST}} = T_{\text{NR}} \Omega^{-2} \mathcal{L}_{\text{PST}}^{\text{div}} + T_{\text{NR}} \mathcal{L}_{\text{PST}}^{\text{fn}} + O(\Omega^4) \quad (9.20)$$

with

$$\mathcal{L}_{\text{PST}}^{\text{div}} = \sqrt{s \det g_0} d^6 \xi, \quad (9.21)$$

$$\mathcal{L}_{\text{PST}}^{\text{fn}} = \sqrt{s \det g_0} d^6 \left[\frac{1}{2} g_0^{ij} (g_2)_{ij} + \frac{1}{2} (\mathcal{H}_1^-)_{ij} (\mathcal{H}_1^*)^{ij} \right] \quad (9.22)$$

where g_0 and g_2 are given in (5.18) and (5.19), and \mathcal{H}_1^- is defined as

$$\mathcal{H}_1^- = \frac{c^2}{2} (\mathcal{H}_1 + c^2 \mathcal{H}_1^*). \quad (9.23)$$

Noting that

$$M' \Gamma_{\bar{B}_1 \dots \bar{B}_5} = \Gamma_{\bar{B}_1 \dots \bar{B}_5} M, \quad (9.24)$$

h_7 is expanded as

$$Th_7 = T_{\text{NR}} \Omega^{-2} h_7^{\text{div}} + T_{\text{NR}} h_7^{\text{fn}} + O(\Omega^3) \quad (9.25)$$

with

$$h_7^{\text{div}} = i\sqrt{s} \frac{1}{5!} \mathbf{L}_0^{\bar{A}_1} \dots \mathbf{L}_0^{\bar{A}_5} \bar{L}_{+0} \Gamma_{\bar{A}_1 \dots \bar{A}_5} L_{+0} \quad (9.26)$$

and

$$h_7^{\text{fn}} = h_2^{(7)} - \frac{c}{2} \mathcal{H}_1 h_1^{(4)}, \quad (9.27)$$

$$\begin{aligned} h_2^{(7)} &= \frac{i\sqrt{s}}{5!} \left[\mathbf{L}_0^{\bar{A}_1} \dots \mathbf{L}_0^{\bar{A}_5} \bar{L}_{-1} \Gamma_{\bar{A}_1 \dots \bar{A}_5} L_{-1} + 2 \mathbf{L}_0^{\bar{A}_1} \dots \mathbf{L}_0^{\bar{A}_5} \bar{L}_{+0} \Gamma_{\bar{A}_1 \dots \bar{A}_5} L_{+2} \right. \\ &\quad + 10 \mathbf{L}_0^{\bar{A}_1} \dots \mathbf{L}_0^{\bar{A}_4} \mathbf{L}_1^{\bar{A}_5} \bar{L}_{+0} \Gamma_{\bar{A}_1 \dots \bar{A}_4 \bar{A}_5} L_{-1} + 5 \mathbf{L}_0^{\bar{A}_1} \dots \mathbf{L}_0^{\bar{A}_4} \mathbf{L}_2^{\bar{A}_5} \bar{L}_{+0} \Gamma_{\bar{A}_1 \dots \bar{A}_5} L_{+0} \\ &\quad + 20 \mathbf{L}_0^{\bar{A}_1} \dots \mathbf{L}_0^{\bar{A}_3} \mathbf{L}_1^{\bar{A}_4} \mathbf{L}_1^{\bar{A}_5} \bar{L}_{+0} \Gamma_{\bar{A}_1 \dots \bar{A}_3 \bar{A}_4 \bar{A}_5} L_{+0} \\ &\quad \left. - \delta^{(1,5)} 6\lambda \epsilon_{\bar{a}'_1 \dots \bar{a}'_5 \bar{a}'_6 \bar{a}'_7} \mathbf{L}_0^{\bar{a}'_1} \dots \mathbf{L}_0^{\bar{a}'_5} \mathbf{L}_1^{\bar{a}'_6} \mathbf{L}_1^{\bar{a}'_7} \right], \end{aligned} \quad (9.28)$$

$$\begin{aligned} h_1^{(4)} &= c \left[\mathbf{L}_0^{\bar{A}} \mathbf{L}_0^{\bar{B}} \bar{L}_{+0} \Gamma_{\bar{A}\bar{B}} L_{-1} + \mathbf{L}_0^{\bar{A}} \mathbf{L}_1^{\bar{B}} \bar{L}_{+0} \Gamma_{\bar{A}\bar{B}} L_{+0} \right. \\ &\quad \left. - \delta^{(3,3)} \frac{6\lambda}{3!} \epsilon_{\bar{a}_1 \bar{a}_2 \bar{a}_3 \bar{a}_4} \mathbf{L}_0^{\bar{a}_1} \mathbf{L}_0^{\bar{a}_2} \mathbf{L}_0^{\bar{a}_3} \mathbf{L}_1^{\bar{a}_4} \right]. \end{aligned} \quad (9.29)$$

This implies that the bosonic 6-form $C^{(6)}$ is expanded as

$$TdC^{(6)} = T_{\text{NR}} \Omega^{-2} dC_0^{(6)} + T_{\text{NR}} dC_2^{(6)} + O(\Omega^2), \quad (9.30)$$

$$dC_0^{(6)} = 0, \quad (9.31)$$

$$dC_2^{(6)} = -\frac{6c^2}{5!} \delta^{(1,5)} \epsilon_{\bar{a}'_1 \dots \bar{a}'_5 \bar{a}'_6 \bar{a}'_7} e_0^{\bar{a}'_1} \dots e_0^{\bar{a}'_5} e_1^{\bar{a}'_6} e_1^{\bar{a}'_7} + \frac{c^2}{2} \lambda \delta^{(3,3)} \epsilon_{\bar{a}_1 \dots \bar{a}_3 \bar{a}_4} e_0^{\bar{a}_1} \dots e_0^{\bar{a}_3} e_1^{\bar{a}_4} H_1. \quad (9.32)$$

Since

$$d\mathcal{L}_{\text{PST}}^{\text{div}} = d(\det(\mathbf{L}_0^{\bar{A}})_i d^6 \xi) = -h_7^{\text{div}}, \quad (9.33)$$

the fermionic contribution of $\mathcal{L}_{\text{PST}}^{\text{div}} + \mathcal{L}_{\text{WZ}}^{\text{div}}$ cancels out. The bosonic term of $\mathcal{L}_{\text{PST}}^{\text{div}}$, $\frac{1}{6!} \epsilon_{\bar{A}_0 \dots \bar{A}_5} e_0^{\bar{A}_0} \dots e_0^{\bar{A}_5}$, is deleted by choosing

$$C_0^{(6)} = -\frac{1}{6!} \epsilon_{\bar{A}_0 \dots \bar{A}_5} e_0^{\bar{A}_0} \dots e_0^{\bar{A}_5} \quad (9.34)$$

which satisfies $dC_0^{(6)} = 0$. In summary, we find that the gluing matrix M leads to the consistent non-relativistic limit of the M5-brane in $\text{AdS}_{4/7} \times \text{S}^{7/4}$.

The non-relativistic M5-brane action is composed of $\mathcal{L}_{\text{PST}}^{\text{fin}}$ in (9.22) and

$$\begin{aligned} \mathcal{L}_{\text{WZ}}^{\text{fin}} = c^2 \int_0^1 dt & \left[\left(\frac{2}{5!} \hat{\mathbf{L}}_0^{\bar{A}_1} \dots \hat{\mathbf{L}}_0^{\bar{A}_5} (\hat{L}_{-1} \Gamma_{\bar{A}_1 \dots \bar{A}_5} \theta_- + \hat{L}_{+2} \Gamma_{\bar{A}_1 \dots \bar{A}_5} \theta_+) \right. \right. \\ & + \frac{2}{4!} \hat{\mathbf{L}}_0^{\bar{A}_1} \dots \hat{\mathbf{L}}_0^{\bar{A}_4} \hat{\mathbf{L}}_1^{\bar{A}_5} (\hat{L}_{-1} \Gamma_{\bar{A}_1 \dots \bar{A}_4 \bar{A}_5} \theta_+ + \hat{L}_{+0} \Gamma_{\bar{A}_1 \dots \bar{A}_4 \bar{A}_5} \theta_-) \\ & + \frac{2}{3!} \hat{\mathbf{L}}_0^{\bar{A}_1} \dots \hat{\mathbf{L}}_0^{\bar{A}_3} \hat{\mathbf{L}}_1^{\bar{A}_4} \hat{\mathbf{L}}_1^{\bar{A}_5} \hat{L}_{+0} \Gamma_{\bar{A}_1 \dots \bar{A}_3 \bar{A}_4 \bar{A}_5} \theta_+ \\ & + \left. \frac{2}{4!} \hat{\mathbf{L}}_0^{\bar{A}_1} \dots \hat{\mathbf{L}}_0^{\bar{A}_4} \hat{\mathbf{L}}_2^{\bar{A}_5} \hat{L}_{+0} \Gamma_{\bar{A}_1 \dots \bar{A}_5} \theta_+ \right) \\ & + \frac{1}{2} \left(\hat{\mathbf{L}}_0^{\bar{A}} \hat{\mathbf{L}}_0^{\bar{B}} (\hat{L}_{+0} \Gamma_{\bar{A} \bar{B}} \theta_- + \hat{L}_{-1} \Gamma_{\bar{A} \bar{B}} \theta_+) + 2 \hat{\mathbf{L}}_0^{\bar{A}} \hat{\mathbf{L}}_1^{\bar{B}} \hat{L}_{+0} \Gamma_{\bar{A} \bar{B}} \theta_+ \right) \hat{\mathcal{H}}_1 \Big] \\ & + C_2^{(6)}. \end{aligned} \quad (9.35)$$

The bosonic contribution is

$$\begin{aligned} \int_{\Sigma} C_2^{(6)} & = 6c^2 \lambda \delta^{(1,5)} \int_{\Sigma} \text{vol}_{\Sigma'_5} \epsilon_{\underline{a}' \underline{b}' y^{\underline{a}'}} dy^{\underline{b}'} + 3c^2 \lambda \delta^{(3,3)} \int_{\Sigma} \text{vol}_{\Sigma_3} y H_1 \\ & = 3i \sqrt{s} \lambda \int d^6 \xi \sqrt{s \det g_0} \left[2 \delta^{(1,5)} \epsilon_{\underline{a}' \underline{b}' y^{\underline{a}'}} \partial_{\xi} y^{\underline{b}'} - \delta^{(3,3)} \partial_{i'} y (^* B_1)^{i'} \right] \end{aligned} \quad (9.36)$$

where ξ and i' represent coordinates on Σ_1 and Σ'_3 respectively, and y is the transverse direction in AdS_4 or S^4 . $*$ means the Hodge dual in Σ'_3 .

Let us fix the κ -symmetry by $\theta_+ = 0$. The θ -dependent term in \mathcal{H} disappears and so we have $\mathcal{H}_1 = H_1$ in this gauge. The t -integration is easily done, and the action is drastically simplified as

$$\begin{aligned} \mathcal{L}_{\text{PST}}^{\text{fin}} & = d^6 \xi \sqrt{s \det g_0} \left[-\bar{\theta}_- \gamma^i D_i \theta_- + \frac{1}{2} g_0^{ij} \partial_i y^A \partial_j y^B \eta_{AB} \right. \\ & \quad \left. + \frac{1}{2} (H_1^-)_{ij} (H_1^*)^{ij} + \frac{\epsilon^2 \lambda^2}{2} (4m y^2 - n y'^2) \right], \end{aligned} \quad (9.37)$$

$$\begin{aligned} \mathcal{L}_{\text{WZ}}^{\text{fin}} & = \int_0^1 dt c^2 \frac{2}{5!} e_0^{\bar{A}_1} \dots e_0^{\bar{A}_5} D(t \bar{\theta}_-) \Gamma_{\bar{A}_1 \dots \bar{A}_5} \theta_- + C_2^{(6)} \\ & = d^6 \xi \sqrt{s \det g_0} \left[-\bar{\theta}_- \gamma^i D_i \theta_- \right] + C_2^{(6)}. \end{aligned} \quad (9.38)$$

Combining these results, we obtain the non-relativistic M5-brane action

$$\begin{aligned}
 S_{\text{NR}} = T_{\text{NR}} \int d^6 \xi \sqrt{s \det g_0} & \left[\frac{1}{2} g_0^{ij} \partial_i y^A \partial_j y^B \eta_{AB} + \frac{\epsilon^2 \lambda^2}{2} (4m y^2 - n y'^2) \right. \\
 & \left. + \frac{1}{2} (H_1^-)_{ij} (H_1^*)^{ij} - 2\bar{\theta}_- \gamma^i D_i \theta_- \right] + T_{\text{NR}} \int_{\Sigma} C_2^{(6)}. \tag{9.39}
 \end{aligned}$$

In the flat limit $\lambda \rightarrow 0$, it is reduced to the linearized M5-brane action considered in [52].

10. Summary and discussions

We have derived the NH superalgebra for AdS branes as IW contractions of the super-AdS \times S algebras in ten- and eleven-dimensions. Requiring that the isometry on the AdS brane worldvolume and the Lorentz symmetry in the transverse space extend to the super-isometry, we classified possible branes. The NH superalgebra contains the super-isometry as a super-subalgebra: $su(2|2) \times su(2|2)$, $osp(4|4)$, $osp(6|2) \times psu(2|1)$ and variants of them for non-relativistic AdS branes in $AdS_5 \times S^5$, and $osp(4|2) \times osp(4|2)$, $osp(6|2) \times so(2|2)$, $sp(4|2) \times osp(4|2)$, $osp(8|2) \times su(2)$ and variants of them for non-relativistic AdS M-branes in $AdS_{4/7} \times S^{7/4}$. The possible branes are summarized in table 1 and 4. These contain 1/2 BPS branes obtained by examining an open superstring in $AdS_5 \times S^5$ and an open supermembrane in $AdS_{4/7} \times S^{7/4}$. We applied the similar analyses to branes in IIB pp-wave and M pp-wave. The possible branes are summarized in table 2, 3, 5 and 6 and we derived the NH superalgebras of these pp-wave branes. It is interesting to apply our procedure to more general cases such as [53].

The WZ terms of AdS branes in ten- and eleven-dimensions are examined by using the CE cohomology on the super-AdS \times S algebras. We find that WZ terms of the AdS branes in $AdS_5 \times S^5$ and $AdS_{4/7} \times S^{7/4}$ are non-trivial elements of the CE cohomology except for those of strings in $AdS_5 \times S^5$.

By taking the non-relativistic limit of the relativistic brane actions obtained above, we derived non-relativistic Dp -brane actions in $AdS_5 \times S^5$ and non-relativistic M-brane actions in $AdS_{4/7} \times S^{7/4}$. We have seen that there exists the consistent non-relativistic limit for $Dp(\text{even}, \text{even})$ for $p = 1 \pmod{4}$ and $Dp(\text{odd}, \text{odd})$ for $p = 3 \pmod{4}$ in $AdS_5 \times S^5$, and $M2(0,3)$, $M2(2,1)$, $M5(1,5)$ and $M5(3,3)$ in $AdS_4 \times S^7$ and $S^4 \times AdS_7$. We derived the non-relativistic actions for these branes.

In the flat limit, the non-relativistic AdS Dp - and M2-brane actions are reduced to non-relativistic flat brane actions [12, 11]. The non-relativistic AdS M5-brane action is reduced to the linearized M5-brane action [52]. It is interesting to examine these non-relativistic AdS brane actions further, but is left for future investigations.

It is also interesting to examine the non-relativistic limit of branes in the pp-wave. It is known that the pp-wave superalgebra is an IW contraction of the AdS superalgebra. So, the brane actions in the pp-wave can be derived from those in the AdS background by expanding supercurrents with respect to the contraction parameter Λ as was presented in appendix C. Once having derived the brane action in the pp-wave one can easily extract the non-relativistic brane actions. These actions can be also derived from the non-relativistic

actions derived in the present paper by expanding supercurrents with respect to the contraction parameter Ω . We hope to report these points elsewhere in near future.

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A. LI Cartan one forms

A.1 $\text{AdS}_5 \times S^5$

Supervielbeins on the $\text{AdS}_5 \times S^5$ can be obtained via the coset construction with the coset supermanifold:

$$\text{AdS}_5 \times S^5 \sim \frac{PSU(2, 2|4)}{SO(1, 4) \times SO(4)}. \quad (\text{A.1})$$

We parametrize the group manifold as

$$g = g_x g_\theta, \quad g_\theta = e^{Q\theta}, \quad Q = (Q_1, Q_2), \quad \theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}. \quad (\text{A.2})$$

where g_x is concretely specified later. The supervielbeins \mathbf{L}^A and L^α , and super spin connection \mathbf{L}^{AB} are the LI Cartan one forms defined by

$$g^{-1}dg = \mathbf{L}^A P_A + \frac{1}{2} \mathbf{L}^{AB} J_{AB} + Q_\alpha L^\alpha, \quad (\text{A.3})$$

$$g_x^{-1}dg_x = e^A P_A + \frac{1}{2} \omega^{AB} J_{AB}, \quad (\text{A.4})$$

where e^A and ω^{AB} are the vielbein and the spin connection of the $\text{AdS}_5 \times S^5$. After some algebra, we obtain⁶

$$\mathbf{L}^A = e^A + 2i \sum_{n=1}^{\infty} \bar{\theta} \Gamma^A \frac{\mathcal{M}^{2n-2}}{(2n)!} D\theta = e^A + 2i \bar{\theta} \Gamma^A \left(\frac{\cosh \mathcal{M} - 1}{\mathcal{M}^2} \right) D\theta, \quad (\text{A.5})$$

$$L^\alpha = \sum_{n=0}^{\infty} \frac{\mathcal{M}^{2n}}{(2n+1)!} D\theta = \frac{\sinh \mathcal{M}}{\mathcal{M}} D\theta, \quad (\text{A.6})$$

$$\mathbf{L}^{AB} = \omega^{AB} - 2i \lambda \bar{\theta} \hat{\Gamma}^{AB} i \sigma_2 \sum_{n=1}^{\infty} \frac{\mathcal{M}^{2n-2}}{(2n)!} D\theta = \omega^{AB} - 2i \lambda \bar{\theta} \hat{\Gamma}^{AB} i \sigma_2 \frac{\cosh \mathcal{M} - 1}{\mathcal{M}^2} D\theta \quad (\text{A.7})$$

⁶The differential d acts as $d(F \wedge G) = dF \wedge G + (-1)^f F \wedge dG$ (where f is the degree of F), and commutes with θ .

with

$$\mathcal{M}^2 = i\lambda \left(\widehat{\Gamma}_A i\sigma_2 \theta \bar{\theta} \Gamma^A - \frac{1}{2} \Gamma_{AB} \theta \bar{\theta} \widehat{\Gamma}^{AB} i\sigma_2 \right), \quad (\text{A.8})$$

$$D\theta = d\theta + \frac{\lambda}{2} e^A \widehat{\Gamma}_A i\sigma_2 \theta + \frac{1}{4} \omega^{AB} \Gamma_{AB} \theta, \quad (\text{A.9})$$

$$\widehat{\Gamma}_A = (-\Gamma_a \mathcal{I}, \Gamma_{a'} \mathcal{J}), \quad \widehat{\Gamma}_{AB} = (-\Gamma_{ab} \mathcal{I}, \Gamma_{a'b'} \mathcal{J}). \quad (\text{A.10})$$

The bosonic subalgebra is a direct product of $\mathfrak{so}(2,4)$ and $\mathfrak{so}(6)$, and so we may consider these parts separately. For an (m, n) -brane, it is convenient to parametrize the group manifold on the $\mathfrak{so}(2,4)$ algebra as

$$g_{AdS} = g_N e^{y^a P_a}, \quad g_N = e^{x^{\bar{a}m} P_{\bar{a}m}} \dots e^{x^{\bar{a}1} P_{\bar{a}1}}. \quad (\text{A.11})$$

For this parametrization, we obtain

$$\begin{aligned} e^{\bar{a}\ell} &= e_N^{\bar{a}\ell} \cosh r_y, \quad \ell = 1, 2, \dots, m, \\ e^a &= \left(\frac{\sinh Y}{Y} dy \right)^a, \\ \omega^{\bar{a}_k \bar{a}\ell} &= \omega_N^{\bar{a}_k \bar{a}\ell}, \\ \omega^{\underline{a}\bar{a}\ell} &= -\lambda^2 y^{\underline{a}} \frac{\sinh r_y}{r_y} e_N^{\bar{a}\ell}, \\ \omega^{ab} &= -2\lambda^2 y^{\underline{a}} \left(\frac{\cosh Y - 1}{Y^2} dy \right)^{\underline{b}], \end{aligned} \quad (\text{A.12})$$

where $r_y^2 = \lambda^2 y^{\underline{a}} y^{\underline{b}} \eta_{\underline{a}\underline{b}} = \lambda^2 y^2$ and $(Y^2)^{\underline{a}\underline{b}} = \lambda^2 (y^2 \delta_{\underline{b}}^{\underline{a}} - y^{\underline{a}} y_{\underline{b}})$. e_N and ω_N defined by

$$g_N^{-1} dg_N = e_N^{\bar{a}} P_{\bar{a}} + \frac{1}{2} \omega_N^{\bar{a}\bar{b}} J_{\bar{a}\bar{b}} \quad (\text{A.13})$$

are obtained as

$$\begin{aligned} e_N^{\bar{a}\ell} &= \cosh r_1 \cdots \cosh r_{\ell-1} dx^{\bar{a}\ell}, \\ \omega_N^{\bar{a}_k \bar{a}\ell} &= -\lambda^2 x^{\bar{a}_k} \frac{\sinh r_k}{r_k} \cosh r_{k+1} \cdots \cosh r_{\ell-1} dx^{\bar{a}\ell}, \quad k < \ell \end{aligned} \quad (\text{A.14})$$

where $r_\ell^2 = \lambda^2 x^{\bar{a}\ell} x^{\bar{a}\ell} \eta_{\bar{a}\bar{a}\ell}$.

The vielbein $e^{a'}$ and the spin connection $\omega^{a'b'}$ of S^5 are obtained as those of AdS_5 with the replacement

$$\lambda^2 \rightarrow -\lambda^2, \quad \bar{a} \rightarrow \bar{a}', \quad \underline{a} \rightarrow \underline{a}', \quad m \rightarrow n. \quad (\text{A.15})$$

Under the scaling with Ω defined in (5.1), the above vielbeins and spin connections are expanded as

$$e^{\bar{a}\ell} = e_0^{\bar{a}\ell} + \Omega^2 e_2^{\bar{a}\ell} + O(\Omega^4), \quad e_0^{\bar{a}\ell} = e_N^{\bar{a}\ell}, \quad e_2^{\bar{a}\ell} = e_N^{\bar{a}\ell} \frac{1}{2} r_y^2, \quad (\text{A.16})$$

$$e^a = \Omega e_1^a + O(\Omega^3), \quad e_1^a = dy^a \quad (\text{A.17})$$

$$\omega^{\bar{a}_k \bar{a}\ell} = \omega_0^{\bar{a}_k \bar{a}\ell}, \quad \omega_0^{\bar{a}_k \bar{a}\ell} = \omega_N^{\bar{a}_k \bar{a}\ell}, \quad (\text{A.18})$$

$$\omega^{\underline{a}\bar{a}\ell} = -\Omega \lambda^2 y^{\underline{a}} e_N^{\bar{a}\ell} + O(\Omega^3), \quad (\text{A.19})$$

$$\omega^{ab} = -2\Omega^2 \lambda^2 y^{\underline{a}} dy^{\underline{b}] + O(\Omega^4). \quad (\text{A.20})$$

A.2 AdS_{4/7} × S^{7/4}

Supervielbeins on the AdS_{4/7} × S^{7/4} can be obtained via the coset construction with the coset supermanifolds:

$$\text{AdS}_4 \times S^7 \sim \frac{OSp(8|4)}{\text{SO}(3,1) \times \text{SO}(7)}, \quad \text{AdS}_7 \times S^4 \sim \frac{OSp(8^*|4)}{\text{SO}(6,1) \times \text{SO}(4)}. \quad (\text{A.21})$$

Parametrizing the manifolds as $g(X, \theta) = g_x e^{\theta Q}$, we obtain the expression of supervielbeins:

$$\mathbf{L}^A = e^A - 2\bar{\theta}\Gamma^A \sum_{n=1} \frac{\mathcal{M}^{2n-2}}{(2n)!} D\theta, \quad (\text{A.22})$$

$$\mathbf{L}^{AB} = \omega^{AB} + 2\lambda\bar{\theta}\hat{\Gamma}^{AB} \sum_{n=1} \frac{\mathcal{M}^{2n-2}}{(2n)!} D\theta, \quad (\text{A.23})$$

$$L = \sum_{n=0} \frac{\mathcal{M}^{2n}}{(2n+1)!} D\theta, \quad (\text{A.24})$$

where we have introduced the following quantities:

$$\begin{aligned} (D\theta)^{\bar{\alpha}} &\equiv d\theta + \frac{\lambda}{2} e^A \hat{\Gamma}_A \theta + \frac{1}{4} \omega^{AB} \Gamma_{AB} \theta, \\ \mathcal{M}^2 &= \lambda(\hat{\Gamma}^A \theta \bar{\theta} \Gamma_A + \frac{1}{2} \Gamma^{AB} \theta \bar{\theta} \hat{\Gamma}_{AB}), \\ \hat{\Gamma}_A &= (2\mathcal{I}\Gamma_a, \mathcal{I}\Gamma_{a'}), \quad \hat{\Gamma}_{AB} = (2\mathcal{I}\Gamma_{ab}, \mathcal{I}\Gamma_{a'b'}). \end{aligned}$$

Here e_M^A and ω_M^{AB} are the vielbein and the spin connection, respectively.

Since the bosonic subalgebra is the direct product of so(3,2) (so(5)) and so(8)(so(6,2)), we may consider these parts separately as in the case of AdS₅ × S⁵. For the former group manifold, a group element is represented by

$$g_4 = g_N e^{y^a P_a}, \quad g_N = e^{x^{\bar{a}m} P_{\bar{a}m}} \dots e^{x^{\bar{a}1} P_{\bar{a}1}}. \quad (\text{A.25})$$

It is straightforward to derive

$$e^{\bar{a}\ell} = e_N^{\bar{a}\ell} \cosh r_y, \quad \ell = 1, \dots, m \quad (\text{A.26})$$

$$e^{\underline{a}} = \left(\frac{\sinh Y}{Y} dy \right)^{\underline{a}}, \quad (\text{A.27})$$

$$\omega^{\bar{a}k\bar{a}\ell} = \omega_N^{\bar{a}k\bar{a}\ell}, \quad (\text{A.28})$$

$$\omega^{\underline{a}b} = -8\epsilon^2 \lambda^2 y^{\underline{a}} \left(\frac{\cosh Y - 1}{Y} dy \right)^{\underline{b}}, \quad (\text{A.29})$$

$$\omega^{\bar{a}\ell\underline{b}} = 4\epsilon^2 \lambda^2 e_N^{\bar{a}\ell} y^{\underline{b}} \frac{\sinh r_y}{r_y} \quad (\text{A.30})$$

with

$$e_N^{\bar{a}\ell} = dx^{\bar{a}\ell} \cosh r_1 \cdots \cosh r_{\ell-1}, \quad (\text{A.31})$$

$$\omega_N^{\bar{a}k\bar{a}\ell} = -4\epsilon^2 \lambda^2 x^{\bar{a}k} dx^{\bar{a}\ell} \frac{\sinh r_k}{r_k} \cosh r_{k+1} \cdots \cosh r_{\ell-1}, \quad k < \ell, \quad (\text{A.32})$$

where $r_k^2 = 4\epsilon^2\lambda^2 x^{\bar{a}_k} x^{\bar{a}_k} \eta_{\bar{a}_k \bar{a}_k}$, $r_y^2 = 4\epsilon^2\lambda^2 y^a y^b \eta_{ab} = 4\epsilon^2\lambda^2 y^2$ and $Y^2 = 4\epsilon^2\lambda^2 (y^2 \delta_{\underline{b}}^{\underline{a}} - y^{\underline{a}} y_{\underline{b}})$. For the latter group manifold, the vielbein and the spin connection can be obtained from those for the former case with the replacement

$$\epsilon^2 \rightarrow -\frac{1}{4}\epsilon^2, \quad \bar{a} \rightarrow \bar{a}', \quad \underline{a} \rightarrow \underline{a}', \quad m \rightarrow n. \quad (\text{A.33})$$

Under the Ω -scaling (9.1), these scale as

$$e^{\bar{a}} = e_0^{\bar{a}} + \Omega^2 e_2^{\bar{a}} + O(\Omega^4), \quad e_0^{\bar{a}} = e_N^{\bar{a}}, \quad e_2^{\bar{a}} = e_N^{\bar{a}} \frac{r_y^2}{2} \quad (\text{A.34})$$

$$e^{\underline{a}} = \Omega e_1^{\underline{a}} + O(\Omega^3), \quad e_1^{\underline{a}} = dy^{\underline{a}}, \quad (\text{A.35})$$

$$\omega^{\bar{a}\bar{b}} = \omega_0^{\bar{a}\bar{b}}, \quad \omega_0^{\bar{a}\bar{b}} = \omega_N^{\bar{a}\bar{b}}, \quad (\text{A.36})$$

$$\omega^{\underline{a}\underline{b}} = O(\Omega^2), \quad (\text{A.37})$$

$$\omega^{\bar{a}\underline{b}} = \Omega 4\epsilon^2\lambda^2 e_N^{\bar{a}} y^{\underline{b}} + O(\Omega^3). \quad (\text{A.38})$$

B. κ -symmetry

B.1 D-branes in $\text{AdS}_5 \times \text{S}^5$

Here we recall the κ -variation of the action (4.1) by following [36, 40, 41]. Here we consider both Lorentzian branes and Euclidean branes.

Following (2.6), one can derive a variation of the supercurrents by using the homotopy formula as follows:

$$\begin{aligned} \delta \mathbf{L}^A &= d\delta x^A + \eta_{BC} \mathbf{L}^B \delta x^{CA} + \eta_{BC} \mathbf{L}^{AB} \delta x^C - 2i \bar{L} \Gamma^A \delta \theta, \\ \delta L &= d\delta \theta - \frac{\lambda}{2} \delta x^A \hat{\Gamma}_A i \sigma_2 L + \frac{\lambda}{2} \mathbf{L}^A \hat{\Gamma}_A i \sigma \delta \theta, \\ \delta \mathbf{L}^{ab} &= d\delta x^{ab} - 2\lambda^2 \mathbf{L}^a \delta x^b + 2\eta_{cd} \mathbf{L}^{ac} \delta x^{db} + 2i\lambda \bar{L} \hat{\Gamma}^{ab} i \sigma_2 \delta \theta, \\ \delta \mathbf{L}^{a'b'} &= d\delta x^{a'b'} + 2\lambda^2 \mathbf{L}^{a'} \delta x^{b'} + 2\eta_{c'd'} \mathbf{L}^{a'c'} \delta x^{d'b'} + 2i\lambda \bar{L} \hat{\Gamma}^{a'b'} i \sigma_2 \delta \theta, \end{aligned} \quad (\text{B.1})$$

where

$$\delta x^A = \delta Z^{\hat{M}} \mathbf{L}_{\hat{M}}^A, \quad \delta x^{AB} = \delta Z^{\hat{M}} \mathbf{L}_{\hat{M}}^{AB}, \quad \delta \theta^\alpha = \delta Z^{\hat{M}} L_{\hat{M}}^\alpha. \quad (\text{B.2})$$

A universal feature of the κ -variation is

$$\delta_\kappa x^A = 0. \quad (\text{B.3})$$

By using (B.1), one can find that

$$\delta_\kappa g_{ij} = -4i \mathbf{L}_{(i}^A \bar{L}_{j)} \Gamma_A \delta \theta \quad (\text{B.4})$$

and

$$\delta_\kappa d\mathcal{F} = -2id(\mathbf{L}^A \bar{L} \Gamma_A \sigma \delta \theta) \quad \rightarrow \quad \delta_\kappa \mathcal{F} = -2i \mathbf{L}^A \bar{L} \Gamma_A \sigma \delta \theta, \quad (\text{B.5})$$

where the exact term is deleted by $\delta_\kappa A$. By using (B.1) and (B.5), we find that

$$\begin{aligned} \delta_\kappa h_{p+2} &= d[\mathcal{C}_\kappa \wedge e^{\mathcal{F}}]_{p+1}, \quad \mathcal{C}_\kappa = \bigoplus_{\ell=\text{even}} \mathcal{C}_\kappa^{(\ell)}, \\ \mathcal{C}_\kappa^{(2n)} &= \frac{2\sqrt{s}}{(2n-1)!} \mathbf{L}^{A_1} \dots \mathbf{L}^{A_{2n-1}} \bar{L} \Gamma_{A_1 \dots A_{2n-1}} (\sigma)^n i\sigma_2 \delta_\kappa \theta \end{aligned} \quad (\text{B.6})$$

so that

$$\delta_\kappa \mathcal{L}_{WZ} = [\mathcal{C}_\kappa \wedge e^{\mathcal{F}}]_{p+1}, \quad (\text{B.7})$$

where $[\bullet]_{p+1}$ represents the $(p+1)$ -form part of \bullet .

By using these expressions, one finds that the action (4.1) is invariant under

$$\delta_\kappa \theta = (1 + \Gamma) \kappa, \quad (\text{B.8})$$

$$\begin{aligned} \Gamma &= \frac{s\sqrt{-s}}{\sqrt{s} \det(g + \mathcal{F})} \sum_{n=0} \frac{1}{2^n n!} \gamma^{j_1 k_1 \dots j_n k_n} \mathcal{F}_{j_1 k_1} \dots \mathcal{F}_{j_n k_n} \\ &\quad \times (-1)^n (\sigma)^{n - \frac{p-3}{2}} i\sigma_2 \frac{1}{(p+1)!} \epsilon^{i_1 \dots i_{p+1}} \gamma_{i_1 \dots i_{p+1}}, \end{aligned} \quad (\text{B.9})$$

where $\gamma_i = \mathbf{L}_i^A \Gamma_A$.

Under the Ω -scaling, Γ is expanded as

$$\Gamma = \Gamma_0 + O(\Omega), \quad (\text{B.10})$$

$$\begin{aligned} \Gamma_0 &= \frac{s\sqrt{-s}}{\sqrt{s} \det g_0} \frac{1}{(p+1)!} \epsilon^{i_1 \dots i_{p+1}} (\mathbf{L}_0^{\bar{A}_0})_{i_1} \dots (\mathbf{L}_0^{\bar{A}_p})_{i_{p+1}} \Gamma_{\bar{A}_0 \dots \bar{A}_p} (\sigma)^{-\frac{p-3}{2}} i\sigma_2 \\ &= M. \end{aligned} \quad (\text{B.11})$$

Expanding κ as

$$\kappa = \kappa_+ + \Omega \kappa_-, \quad \kappa_\pm = \mathcal{P}_\pm \kappa_\pm \quad (\text{B.12})$$

leads to

$$\delta_\kappa \theta_+ = (1 + \Gamma_0) \kappa_+ = 2\kappa_+. \quad (\text{B.13})$$

This implies that the κ -symmetry can be gauge fixed by choosing $\theta_+ = 0$ since $\delta_\kappa \theta_+|_{\theta_+=0} = 2\kappa_+$ [15].

For an F-string, we obtain the similar expression with $\sigma = -\sigma_1$ and $\mathcal{F} = 0$. Hence the action is κ -invariant, and the κ -gauge symmetry is fixed by $\theta_+ = 0$.

B.2 M-branes in $\text{AdS}_{4/7} \times \mathbf{S}^{7/4}$

Following [48, 49, 51], we recall the κ -symmetry of the M-brane actions. Here we shall consider Euclidean branes as well as Lorentzian branes.

A variation of the supercurrents is derived from (6.5) as

$$\begin{aligned}
 \delta \mathbf{L}^A &= d\delta x^A - \eta_{BC}\delta x^{AB}\mathbf{L}^C + \eta_{BC}\mathbf{L}^{AB}\delta x^C + 2\bar{L}\Gamma^A\delta\theta, \\
 \delta \mathbf{L}^{ab} &= d\delta x^{ab} + 8\epsilon^2\lambda^2\mathbf{L}^a\delta x^b + 2\eta_{cd}\mathbf{L}^{ca}\delta x^{bd} - 2\lambda\bar{L}\hat{\Gamma}^{ab}\delta\theta, \\
 \delta \mathbf{L}^{a'b'} &= d\delta x^{a'b'} - 2\epsilon^2\lambda^2\mathbf{L}^{a'}\delta x^{b'} + 2\eta_{c'd'}\mathbf{L}^{c'a'}\delta x^{b'd'} - 2\lambda\bar{L}\hat{\Gamma}^{a'b'}\delta\theta, \\
 \delta L &= d\delta\theta + \frac{1}{2}\delta x^A\hat{\Gamma}_A L - \frac{\lambda}{2}\mathbf{L}^A\hat{\Gamma}_A\delta\theta - \frac{1}{4}\delta x^{AB}\Gamma_{AB}L + \frac{1}{4}\mathbf{L}^{AB}\Gamma_{AB}\delta\theta, \quad (\text{B.14})
 \end{aligned}$$

where δx^A , δx^{AB} and $\delta\theta$ have been defined in (B.2). For the κ -variation, we require that $\delta_\kappa x^A = 0$.

B.2.1 M2-brane

Let us first consider the case of an M2-brane. From (B.14), one can obtain

$$\delta_\kappa g_{ij} = 4\mathbf{L}^A_{(i}\bar{L}_{j)}\Gamma_A\delta_\kappa\theta, \quad (\text{B.15})$$

and

$$\delta_\kappa h_4 = d(-c\mathcal{L}^A\mathbf{L}^B\bar{L}\Gamma_{AB}\delta_\kappa\theta) \quad \rightarrow \quad \delta_\kappa\mathcal{L}_{\text{WZ}}^{M2} = -c\mathcal{L}^A\mathbf{L}^B\bar{L}\Gamma_{AB}\delta_\kappa\theta. \quad (\text{B.16})$$

By using them one can see that the action (8.1) is invariant under

$$\delta_\kappa\theta = (1 + \Gamma)\kappa, \quad \Gamma = \frac{i\sqrt{s}}{\sqrt{s}\det g} \frac{1}{3!}\epsilon^{ijk}\gamma_{ijk}. \quad (\text{B.17})$$

Under Ω -scaling, Γ is expanded as

$$\Gamma = \Gamma_0 + O(\Omega), \quad \Gamma_0 = i\sqrt{s}\Gamma_{\bar{A}_0\bar{A}_1\bar{A}_2} = M. \quad (\text{B.18})$$

By expanding κ as (B.12), we derive

$$\delta_\kappa\theta_+ = (1 + \Gamma_0)\kappa_+ = 2\kappa_+ \quad (\text{B.19})$$

which implies that the κ -gauge symmetry is fixed by $\theta_+ = 0$.

B.2.2 M5-brane

Next, we consider the case of an M5-brane. A variation of \mathcal{H} is taken with (B.14) as follows:

$$-c\delta_\kappa d\mathcal{H} = \delta_\kappa h_4 = d(-c\mathbf{L}^A\mathbf{L}^B\bar{L}\Gamma_{AB}\delta_\kappa\theta) \quad \rightarrow \quad \delta_\kappa\mathcal{H} = \mathbf{L}^A\mathbf{L}^B\bar{L}\Gamma_{AB}\delta_\kappa\theta, \quad (\text{B.20})$$

where the exact term is deleted by $\delta_\kappa B$. Noting that

$$\delta_\kappa h_7 = c^2 d\left[\frac{2}{5!}\mathbf{L}^{A_1}\dots\mathbf{L}^{A_5}\bar{L}\Gamma_{A_1\dots A_5}\delta_\kappa\theta + \frac{1}{2}\mathbf{L}^A\mathbf{L}^B\bar{L}\Gamma_{AB}\delta_\kappa\theta\mathcal{H}\right], \quad (\text{B.21})$$

we see that

$$\delta_\kappa\mathcal{L}_{\text{WZ}} = c^2\left[\frac{2}{5!}\mathbf{L}^{A_1}\dots\mathbf{L}^{A_5}\bar{L}\Gamma_{A_1\dots A_5}\delta_\kappa\theta + \frac{1}{2}\mathbf{L}^A\mathbf{L}^B\bar{L}\Gamma_{AB}\delta_\kappa\theta\mathcal{H}\right], \quad (\text{B.22})$$

where $c^2 = i\sqrt{s}$. The κ -invariance of the action is shown by following [51] (see also [52]). By using the expressions derived above and the following useful relations

$$\begin{aligned}
 \bar{\gamma} &= \frac{s\sqrt{-s}}{6!\sqrt{s \det g}} \epsilon^{i_1 \dots i_6} \gamma_{i_1 \dots i_6}, \quad \bar{\gamma}^2 = 1, \\
 \epsilon^{i_1 \dots i_6 - n j_1 \dots j_n} \gamma_{j_1 \dots j_n} &= (-1)^{\lfloor \frac{6-n}{2} \rfloor} n! \frac{s\sqrt{-s}}{\sqrt{s \det g}} \gamma^{i_1 \dots i_6 - n} \bar{\gamma}, \\
 \epsilon^{i_1 \dots i_6 - n k_1 \dots k_n} \epsilon_{j_1 \dots j_6 - n k_1 \dots k_n} &= sn!(6-n)! \delta_{[j_1}^{i_1} \dots \delta_{j_6-n]}^{i_6-n}, \\
 \mathcal{H}_{ijk} &= 3\mathcal{H}_{[ij} v_{k]} - \frac{s}{2} \sqrt{s \det g} \epsilon_{ijklmn} \mathcal{H}^{*lm} v^n,
 \end{aligned} \tag{B.23}$$

it is shown that the M5-brane action is invariant under the κ -variation

$$\delta_\kappa \theta = (1 + \Gamma)\kappa, \quad \delta_\kappa a = 0, \tag{B.24}$$

$$\Gamma = \frac{\sqrt{s \det g}}{\sqrt{s \det(g - ic^2 \mathcal{H}^*)}} \left(\bar{\gamma} - \frac{c^2}{2} \mathcal{H}_{ij}^* v_k \gamma^{ijk} - \frac{sc^2}{16\sqrt{s \det g}} \epsilon^{i_1 \dots i_6} \mathcal{H}_{i_1 i_2}^* \mathcal{H}_{i_3 i_4}^* \gamma_{i_5 i_6} \right). \tag{B.25}$$

Under the Ω -scaling, Γ is expanded as

$$\Gamma = \Gamma_0 + O(\Omega), \tag{B.26}$$

$$\Gamma_0 = \frac{s\sqrt{-s}}{\sqrt{s \det g_0}} \frac{1}{6!} \epsilon^{i_1 \dots i_6} (\mathbf{L}_0^{\bar{A}_1})_{i_1} \dots (\mathbf{L}_0^{\bar{A}_6})_{i_6} \Gamma_{\bar{A}_1 \dots \bar{A}_6} = s\sqrt{-s} \Gamma_{\bar{A}_0 \dots \bar{A}_5} = M, \tag{B.27}$$

which implies that

$$\delta_\kappa \theta_+ = (1 + \Gamma_0)\kappa_+ = 2\kappa_+. \tag{B.28}$$

Thus the κ -symmetry is gauge fixed by $\theta_+ = 0$.

C. Penrose limit of brane actions

Here we will construct the action of D-branes and M-branes on pp-wave backgrounds via the Penrose limit, instead of non-relativistic limit. This is a natural application of our procedure. The Penrose limit of an alternative action of an AdS superstring has been discussed in [25]. The result includes Metsaev's results for F-string [54] and D3-brane [55] on the maximally supersymmetric pp-wave.

C.1 Branes in IIB pp-wave

We derive the Dp -brane action in the IIB pp-wave from the Dp -brane action in $\text{AdS}_5 \times \text{S}^5$

$$\begin{aligned}
 S &= T \int \mathcal{L}_{DBI} + \mathcal{L}_{WZ}, \\
 \mathcal{L}_{DBI} &= \sqrt{s \det(g + \mathcal{F})} d^{p+1} \xi, \quad d\mathcal{L}_{WZ} = h_{p+2} = \sum_{n=0} h^{(p+2-2n)} \mathcal{F}^n, \\
 h^{(2n+1)} &= \frac{c}{(2n-1)!} \left[\mathbf{L}^{A_1} \dots \mathbf{L}^{A_{2n-1}} \bar{L} \Gamma_{A_1 \dots A_{2n-1}} \sigma^{n+1} \varrho L \right. \\
 &\quad \left. + \delta_{n,2} \frac{i}{5} \lambda (\epsilon_{a_1 \dots a_5} \mathbf{L}^{a_1} \dots \mathbf{L}^{a_5} - \epsilon_{a'_1 \dots a'_5} \mathbf{L}^{a'_1} \dots \mathbf{L}^{a'_5}) \right]
 \end{aligned} \tag{C.1}$$

where $\sigma = \sigma_3$ and $\varrho = \sigma_1$. $c = \sqrt{s}$ is required by the κ -invariance of the action.

The Penrose limit considered in section 3 is equivalent to scaling the coordinates as

$$X^+ \rightarrow \Lambda^2 X^+, \quad X^{\hat{i}} \rightarrow \Lambda X^{\hat{i}}, \quad \theta_+ \rightarrow \Lambda \theta_+ \quad (\text{C.2})$$

and taking the limit $\Lambda \rightarrow 0$. Under the scaling, LI Cartan one-forms are expanded as

$$\begin{aligned} \mathbf{L}^+ &= \sum_{n=0} \Lambda^{2n+2} \mathbf{L}_{2n+2}^+, & \mathbf{L}^{\hat{i}} &= \sum_{n=0} \Lambda^{2n+1} \mathbf{L}_{2n+1}^{\hat{i}}, & \mathbf{L}_{*}^{\hat{i}} &= \sum_{n=0} \Lambda^{2n+1} \mathbf{L}_{*2n+1}^{\hat{i}}, \\ L_+ &= \sum_{n=0} \Lambda^{2n+1} L_{+2n+1} \end{aligned} \quad (\text{C.3})$$

where

$$\mathbf{L}^{\pm} = \frac{1}{\sqrt{2}} (\mathbf{L}^9 \pm \mathbf{L}^0), \quad L_{\pm} = \ell_{\pm} L_{\pm}, \quad \mathbf{L}_{*}^{\hat{i}} = (\mathbf{L}^{0i}, \mathbf{L}^{9i'}) . \quad (\text{C.4})$$

Under the expansion (C.3), we derive

$$g_{ij} = \Lambda^2 g_{ij}^{(pp)} + O(\Lambda^4), \quad g_{ij}^{(pp)} = 2(\mathbf{L}_2^+)_{i} (\mathbf{L}_0^-)_{j} + (\mathbf{L}_1^{\hat{i}})_{i} (\mathbf{L}_1^{\hat{j}})_{j} \eta_{\hat{i}\hat{j}} . \quad (\text{C.5})$$

It follows from⁷

$$d\mathcal{F} = \Lambda^2 [-i\mathbf{L}^+ \bar{L}_- \Gamma_+ \sigma L_- - i\mathbf{L}^- \bar{L}_+ \Gamma_- \sigma L_+ - 2i\mathbf{L}^{\hat{i}} \bar{L}_+ \Gamma_{\hat{i}} \sigma L_-] \quad (\text{C.6})$$

that

$$\mathcal{F} = \Lambda^2 \mathcal{F}_{pp} + O(\Lambda^4) \quad (\text{C.7})$$

where we assume that $F = \Lambda^2 F_{pp}$. These imply that

$$\mathcal{L}_{\text{DBI}} = \Lambda^{p+1} \mathcal{L}_{\text{DBI}}^{pp} + O(\Lambda^{p+3}), \quad \mathcal{L}_{\text{DBI}}^{pp} = \sqrt{s \det(g_{pp} + \mathcal{F}_{pp})} d^{p+1} \xi . \quad (\text{C.8})$$

The factor Λ^{p+1} is absorbed into the definition of the tension as

$$T = \Lambda^{-(p+1)} T_{pp} . \quad (\text{C.9})$$

One finds that the fermionic part of $h^{(2n+1)}$ is scaled as

$$\begin{aligned} h^{(2n+1)}|_{\text{fermionic}} &= \Lambda^{2n} h_{pp}^{(2n+1)}|_{\text{fermionic}} + O(\Lambda^{2n+2}), \\ h_{pp}^{(2n+1)}|_{\text{fermionic}} &= \frac{c}{(2n-1)!} \mathbf{L}^{A_1} \dots \mathbf{L}^{A_{2n-1}} \bar{L} \Gamma_{A_1 \dots A_{2n-1}} \sigma_3^{n+1} \sigma_1 L \\ &= \frac{c}{(2n-1)!} \mathbf{L}^{\hat{i}_1} \dots \mathbf{L}^{\hat{i}_{2n-1}} (\bar{L}_+ \Gamma_{\hat{i}_1 \dots \hat{i}_{2n-1}} \sigma_3^{n+1} \sigma_1 L_- \\ &\quad + \bar{L}_- \Gamma_{\hat{i}_1 \dots \hat{i}_{2n-1}} \sigma_3^{n+1} \sigma_1 L_+) \\ &\quad + \frac{c}{(2n-2)!} \mathbf{L}^+ \mathbf{L}^{\hat{i}_1} \dots \mathbf{L}^{\hat{i}_{2n-2}} \bar{L}_- \Gamma_{+\hat{i}_1 \dots \hat{i}_{2n-2}} \sigma_3^{n+1} \sigma_1 L_- \\ &\quad + \frac{c}{(2n-2)!} \mathbf{L}^- \mathbf{L}^{\hat{i}_1} \dots \mathbf{L}^{\hat{i}_{2n-2}} \bar{L}_+ \Gamma_{-\hat{i}_1 \dots \hat{i}_{2n-2}} \sigma_3^{n+1} \sigma_1 L_+ \\ &\quad + \frac{c}{(2n-3)!} \mathbf{L}^+ \mathbf{L}^- \mathbf{L}^{\hat{i}_1} \dots \mathbf{L}^{\hat{i}_{2n-3}} (\bar{L}_- \Gamma_{\hat{i}_1 \dots \hat{i}_{2n-3}} \sigma_3^{n+1} \sigma_1 L_+ \\ &\quad - \bar{L}_+ \Gamma_{\hat{i}_1 \dots \hat{i}_{2n-3}} \sigma_3^{n+1} \sigma_1 L_-) . \end{aligned} \quad (\text{C.11})$$

⁷ \mathbf{L}^+ , \mathbf{L}^- , $\mathbf{L}^{\hat{i}}$, L_+ and L_- are understood as \mathbf{L}_2^+ , \mathbf{L}_0^- , $\mathbf{L}_1^{\hat{i}}$, L_{+1} and L_{-0} respectively below.

For the bosonic part, we derive

$$h^{(5)}|_{\text{bosonic}} = \Lambda^4 h_{pp}^{(5)}|_{\text{bosonic}} + O(\Lambda^6), \quad (\text{C.12})$$

$$h_{pp}^{(5)}|_{\text{bosonic}} = -ic \frac{4\lambda}{\sqrt{2}} \mathbf{L}^- (\mathbf{L}^1 \mathbf{L}^2 \mathbf{L}^3 \mathbf{L}^4 + \mathbf{L}^5 \mathbf{L}^6 \mathbf{L}^7 \mathbf{L}^8). \quad (\text{C.13})$$

The $O(\Lambda^6)$ -term which contains \mathbf{L}^+ disappears in the limit.

To summarize the pp-wave Dp-brane action is given as

$$S_{pp} = T_{pp} \int \mathcal{L}_{\text{DBI}}^{pp} + \mathcal{L}_{\text{WZ}}^{pp} \quad (\text{C.14})$$

with (C.8) and

$$h_{p+2}^{pp} = d\mathcal{L}_{\text{WZ}}^{pp} = \sum_{n=0} h_{pp}^{(p+2-2n)} \mathcal{F}_{pp}^n, \quad (\text{C.15})$$

$$h_{pp}^{(2n+1)} = \frac{c}{(2n-1)!} \mathbf{L}^{A_1} \dots \mathbf{L}^{A_{2n-1}} \bar{L} \Gamma_{A_1 \dots A_{2n-1}} \sigma^{n+1} \varrho L - ic \delta_{n,2} \frac{4\lambda}{\sqrt{2}} \mathbf{L}^- (\mathbf{L}^1 \mathbf{L}^2 \mathbf{L}^3 \mathbf{L}^4 + \mathbf{L}^5 \mathbf{L}^6 \mathbf{L}^7 \mathbf{L}^8). \quad (\text{C.16})$$

This reproduces the pp-wave D3-brane action given in [55] as the $p = 3$ case. Let $\varrho = \sigma_3$ and $\sigma = -\sigma_1$ and replace \mathcal{L}_{DBI} with \mathcal{L}_{NG} or with the Polyakov action, then it is reduced to the pp-wave F-string action constructed in [54].

The $(p+2)$ -form h_{p+2}^{pp} can be shown to be a non-trivial element of the CE cohomology except for h_3 by following the procedure explained in section 4.1. It is easy to obtain the $(p+1)$ -dimensional form of the WZ term as was done in section 4.2.

C.2 Branes in M pp-wave

The Penrose limit considered in section 7 is equivalent to scaling the coordinates as (C.2) and taking the limit $\Lambda \rightarrow 0$. Under the scaling, LI Cartan one-forms are expanded as (C.3) where we define Cartan one-forms as

$$\mathbf{L}^\pm = \frac{1}{\sqrt{2}} (\mathbf{L}^\natural \pm \mathbf{L}^0), \quad \mathbf{L}_*^{\hat{i}} = \begin{cases} (\mathbf{L}^{i0}, \mathbf{L}^{i'\natural}) & \text{for } \text{AdS}_4 \times \text{S}^7 \\ (\mathbf{L}^{i\natural}, \mathbf{L}^{i'0}) & \text{for } \text{AdS}_7 \times \text{S}^4 \end{cases}, \quad L_\pm = \ell_\pm L. \quad (\text{C.17})$$

C.2.1 Penrose limit of M2 brane action

We consider the Penrose limit of the M2-brane action

$$S = T \int \mathcal{L}_{\text{NG}} + \mathcal{L}_{\text{WZ}}, \quad (\text{C.18})$$

$$\mathcal{L}_{\text{NG}} = \sqrt{s \det g}, \quad (\text{C.19})$$

$$d\mathcal{L}_{\text{WZ}} = h_4 = c \left[\frac{1}{2} \mathbf{L}^A \mathbf{L}^B \bar{L} \Gamma_{AB} L - \frac{6\lambda}{4!} \epsilon_{a_1 \dots a_4} \mathbf{L}^{a_1} \dots \mathbf{L}^{a_4} \right] \quad (\text{C.20})$$

where $c = i\sqrt{s}$ is required by the κ -invariance of the action. Under the expansion (C.3), \mathcal{L}_{NG} is expanded as

$$\mathcal{L}_{\text{NG}} = \Lambda^3 \mathcal{L}_{\text{NG}}^{pp} + O(\Lambda^5), \quad \mathcal{L}_{\text{NG}}^{pp} = \sqrt{s \det g_{pp}} \quad (\text{C.21})$$

while $d\mathcal{L}_{\text{WZ}} = h_4$ is expanded as

$$h_4 = \Lambda^3 h_4^{pp} + O(\Lambda^5), \quad (\text{C.22})$$

$$h_4^{pp} = c \left[\frac{1}{2} \mathbf{L}^A \mathbf{L}^B \bar{\mathbf{L}} \Gamma_{AB} L + \frac{6\lambda}{\sqrt{2}} \mathbf{L}^- \mathbf{L}^1 \mathbf{L}^2 \mathbf{L}^3 \mathbf{L}^4 \right]. \quad (\text{C.23})$$

The Λ^3 factor is absorbed into the definition of the tension as $T = \Lambda^{-3} T_{pp}$. The pp-wave M2-brane action is given as

$$S = T_{pp} \int \mathcal{L}_{\text{NG}}^{pp} + \mathcal{L}_{\text{WZ}}^{pp} \quad (\text{C.24})$$

with (C.21) and $h_4^{pp} = d\mathcal{L}_{\text{WZ}}^{pp}$ with (C.23).

C.2.2 Penrose limit of M5-brane action

Next we will consider the M5-brane action

$$S = T \int \mathcal{L}_{\text{PST}} + \mathcal{L}_{\text{WZ}}, \quad (\text{C.25})$$

with

$$\mathcal{L}_{\text{PST}} = \sqrt{s \det(g_{ij} - ic^2 \mathcal{H}_{ij}^*)} + c^2 \frac{\sqrt{s \det g}}{4} \mathcal{H}^{*ij} \mathcal{H}_{ij}, \quad (\text{C.26})$$

$$\mathcal{H}_{ij} = \mathcal{H}_{ijk} v^k, \quad \mathcal{H}^{*ij} = \mathcal{H}^{*ijk} v_k, \quad v_i = \frac{\partial_i a}{\sqrt{g^{jk} \partial_j a \partial_k a}},$$

$$\mathcal{H} = H + \mathcal{C}_3, \quad \mathcal{H}^{*ijk} = \frac{1}{3! \sqrt{s \det g}} \epsilon^{ijklmn} \mathcal{H}_{lmn}, \quad H = dB$$

and

$$d\mathcal{L}_{\text{WZ}} = h_7, \quad h_7 = h^{(7)} - \frac{c}{2} h^{(4)} \mathcal{H}, \quad (\text{C.27})$$

$$h^{(4)} = c \left[\frac{1}{2} \mathbf{L}^A \mathbf{L}^B \bar{\mathbf{L}} \Gamma_{AB} L - \frac{6\lambda}{4!} \epsilon_{a_1 \dots a_4} \mathbf{L}^{a_1} \dots \mathbf{L}^{a_4} \right], \quad (\text{C.28})$$

$$h^{(7)} = c^2 \left[\frac{1}{5!} \mathbf{L}^{A_1} \dots \mathbf{L}^{A_5} \bar{\mathbf{L}} \Gamma_{A_1 \dots A_5} L - \frac{6\lambda}{7!} \epsilon_{a'_1 \dots a'_7} \mathbf{L}^{a'_1} \dots \mathbf{L}^{a'_7} \right], \quad (\text{C.29})$$

$$-cd\mathcal{H} = h^{(4)} \quad (\text{C.30})$$

with $c^2 = i\sqrt{s}$ for the κ -invariance of the action.

Observe that under the expansion (C.3),

$$\begin{aligned} a &= a_{pp}, \quad v_i = \Lambda v_i^{pp} + O(\Lambda^3), \quad \mathcal{H}_{ijk} = \Lambda^3 \mathcal{H}_{ijk}^{pp} + O(\Lambda^5), \quad \mathcal{H}_{ij} = \Lambda^2 \mathcal{H}_{ij}^{pp} + O(\Lambda^4), \\ \mathcal{H}^{*ijk} &= \Lambda^{-3} \mathcal{H}_{pp}^{*ijk} + O(\Lambda^{-1}), \quad \mathcal{H}_{ij}^* = \Lambda^2 \mathcal{H}_{ij}^{*pp} + O(\Lambda^4) \end{aligned} \quad (\text{C.31})$$

where we assume that $H = \Lambda^3 H_{pp}$. These imply that

$$\begin{aligned} \mathcal{L}_{\text{PST}} &= \Lambda^6 \mathcal{L}_{\text{PST}}^{pp} + O(\Lambda^8), \\ \mathcal{L}_{\text{PST}}^{pp} &= \sqrt{s \det(g_{ij}^{pp} - ic^2 \mathcal{H}_{ij}^{*pp})} + c^2 \frac{\sqrt{s \det g_{pp}}}{4} \mathcal{H}_{pp}^{*ij} \mathcal{H}_{ij}^{pp}. \end{aligned} \quad (\text{C.32})$$

The WZ term h_7 is expanded as

$$h_7 = \Lambda^6 h_7^{pp}, \quad h_7^{pp} = h_7^{(7)} - \frac{c}{2} h_{pp}^{(4)} \mathcal{H}_{pp}, \quad (\text{C.33})$$

$$h_7^{(7)} = c^2 \left[\frac{1}{5!} \mathbf{L}^{A_1} \dots \mathbf{L}^{A_5} \bar{\mathbf{L}} \Gamma_{A_1 \dots A_5} L - \frac{6\lambda}{\sqrt{2}} \mathbf{L}^{-\mathbf{L}^4} \dots \mathbf{L}^9 \right], \quad (\text{C.34})$$

$$h_{pp}^{(4)} = c \left[\frac{1}{2} \mathbf{L}^A \mathbf{L}^B \bar{\mathbf{L}} \Gamma_{AB} L + \frac{6\lambda}{\sqrt{2}} \mathbf{L}^{-\mathbf{L}^1} \mathbf{L}^2 \mathbf{L}^3 \mathbf{L}^4 \right]. \quad (\text{C.35})$$

The Λ^6 factor is absorbed into the definition of the tension as $T = \Lambda^{-6} T_{pp}$. The pp-wave M5-brane action is given as

$$S = T_{pp} \int \mathcal{L}_{\text{PST}}^{pp} + \mathcal{L}_{\text{WZ}}^{pp} \quad (\text{C.36})$$

with (C.32) and $h_7^{pp} = d\mathcal{L}_{\text{WZ}}^{pp}$ with (C.33).

Following the procedure explained in section 8.1, one can show that the $(p+2)$ -form h_{p+2}^{pp} is a non-trivial element of the CE cohomology. The $(p+1)$ -dimensional form of the WZ term can be obtained easily as was done in section 8.2.

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